Planetary Heating

1. The exponential depth dependence of heat production is preferred because it is self-preserving with erosion. However, many alternative models can be prescribed. One such model is that of a two-layer system with $H = H_1$ and $k = k_1$ for $0 \leq z \leq h_1$, and $H = H_2$ and $k = k_2$ for $h_1 \leq z \leq h_2$. For $z > h_2$, $k = k_3$, $H = 0$ and the upward heat flux is $q_m$.

   (a) Derive the one-dimensional (flat earth) equation for temperature as a function of depth.

   (b) Determine the heat flow at the surface ($z = 0$), and temperatures at $z = h_1$ and $z = h_2$ for $\rho_1 = 2800$ kg m$^{-3}$, $\rho_2 = 3200$ kg m$^{-3}$, $k_1 = k_2 = k_3 = 2.6$ W m$^{-1}$ K$^{-1}$, $h_1 = 8$ km, $h_2 = 40$ km, $\rho_1 H_1 = 2 \mu$W m$^{-3}$, $\rho_2 H_2 = 0.36 \mu$W m$^{-3}$, $T_0 = 0$ °C, and $q_m = 10$ mW m$^{-2}$.

2. Derive an expression for the temperature at the center of a planet of radius $a$ (spherical coordinates) with uniform density $\rho$ and internal heat generation $H$. Heat transfer in the planet is by conduction only in the lithosphere, which extends from $r = b$ to $r = a$. For $0 \leq r \leq b$ heat transfer is by convection, which maintains the temperature gradient $dT/dr$ constant at the adiabatic value $-\Gamma$. The surface temperature is $T_0$. To solve for $T(r)$, you need to assume that $T$ and the heat flux are continuous at $r = b$.

Instantaneous Heating

3. One way of determining the effects of erosion on subsurface temperatures is to consider the instantaneous removal of a layer of material with thickness $l$. Prior to the removal $T(z) = T_0 + \beta z$, where $z$ is the depth, $\beta$ is the geothermal gradient, and $T_0$ is the surface temperature. After removal, the new surface is maintained at temperature $T_0$. Show that the subsurface temperature after the removal of the surface layer is given by:

   $$ T = T_0 + \beta z + \beta l \text{ erf} \left( \frac{z}{\sqrt{\kappa t}} \right). $$

Additional Problem for Graduate Section

Thermal Stress

4. Because of its cooling, the sea-floor subsides relative to the continent at a passive continental margin (non-subducting boundary). Making approximate assumptions about the oceanic lithosphere:

   (a) What is the velocity of subsidence for an oceanic plate of 10 Ma assuming the continental lithosphere remains consistently buoyant?

   (b) Assuming the strain is accommodated linearly over a 10 km wide zone, what is the resultant shear strain rate, and loading stress rate at the boundary?