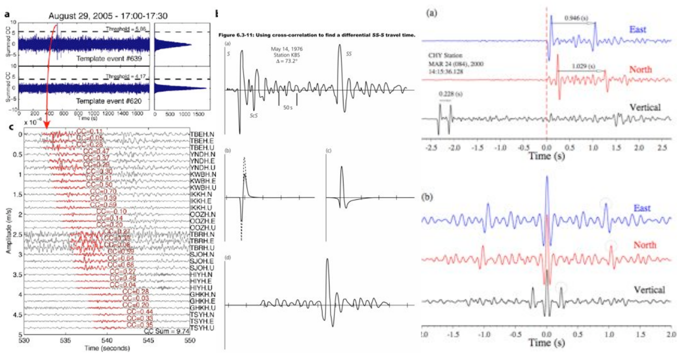
EAS 4803/8803 - Observational Seismology

Lec#3: Linear Systems (cont.)

Dr. Zhigang Peng, Spring 2013



Last Time

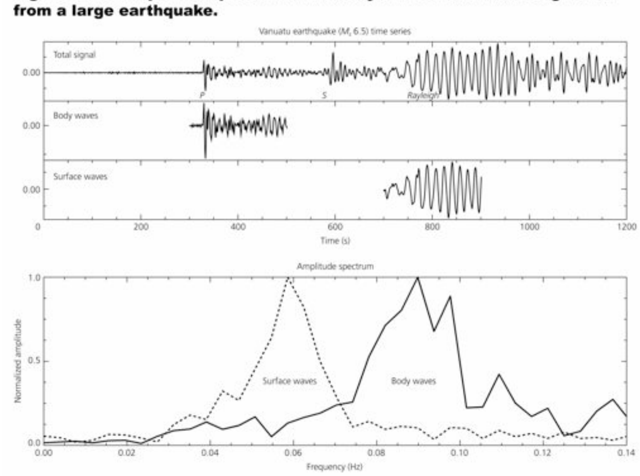
- Linear systems
  - Basic models
  - Convolution and deconvolution modeling
  - Finite length signals
  - Correlation

This Time

- Linear systems
  - Basic models
  - Convolution and deconvolution modeling
  - Finite length signals
  - Correlation
- Discrete time series & transforms

Reading: Stein and Wysession Chap. 6.3-6.4

Figure 6.2-3: Amplitude spectra for the body and surface wave segments from a large earthquake.



Finite length signals

- Consider a window function $b(t)$. Its effect on the data $f(t)$ is represented by multiplying $f(t)$ by $b(t)$.

$$G(\varpi) = \int_{-\infty}^{\infty} b(t) f(t) e^{-i\varpi t} dt$$

$$G(\varpi) = \int_{-\infty}^{\infty} \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} B(\varpi') e^{i\varpi' t} d\varpi' \right] \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} F(\varpi'') e^{i\varpi'' t} d\varpi'' \right] e^{-i\varpi t} dt$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} B(\varpi') \left[\int_{-\infty}^{\infty} F(\varpi'') \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\varpi t + i\varpi' t + i\varpi'' t} dt \right] d\varpi'' \right] d\varpi'$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} B(\varpi') \left[\int_{-\infty}^{\infty} F(\varpi'') \delta(\varpi - \varpi' - \varpi'') d\varpi'' \right] d\varpi'$$

Finite length signals

- Using the sifting properties, we obtain

$$G(\varpi) = \frac{1}{2\pi} \int_{-\infty}^{\infty} B(\varpi') \left[\int_{-\infty}^{\infty} F(\varpi'') \delta(\varpi - \varpi' - \varpi'') d\varpi'' \right] d\varpi'$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} B(\varpi') F(\varpi - \varpi') d\varpi' = \frac{1}{2\pi} B(\varpi) * F(\varpi)$$

- Hence, the effect of multiplying a time series by a window function is that the spectrum of the time series is convolved with the spectrum of the window function.
- This is what is expected!

Effects of a boxcar window function

$b(t) = 1$ for $-T<t<T$,
$= 0$, otherwise.

Its Fourier transform is:

$$B(\varpi) = \int_{-\infty}^{\infty} e^{-i\varpi t}dt = \frac{e^{-i\varpi t}}{-i\varpi}\Big|_{-T}^{T} = \frac{2\sin\varpi T}{\varpi} = \frac{2T\sin\varpi T}{\varpi T}$$

Figure 6.3-8: A boxcar function in the time and frequency domains.

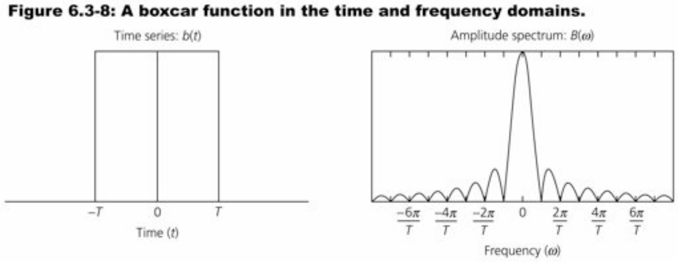
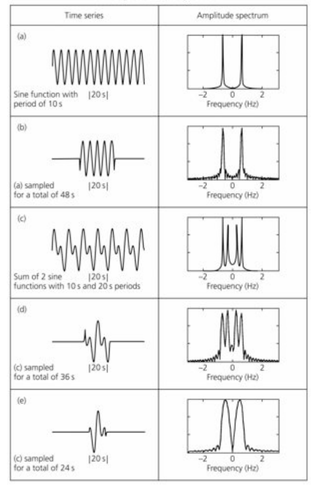


Figure 6.3-9: Effects of windowing time signals on the amplitude spectra.
Data length and frequency resolution



$f(t)$ is a sine wave. What's the effect?

Taking a finite length of record "smears" the delta functions of the infinite length record's spectrum into boader peaks with side lobes.

Input signals contains different frequencies

The frequency resolution, the minimum separation in frequency for which two peaks can be resolved, is proportional to the reciprocal of the total length.

"Uncertainty principle" in time and frequency domains

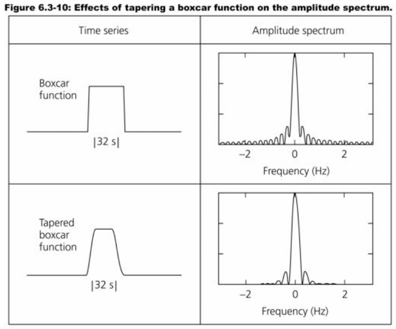
W. Heisenberg

- The product of the "widths" in the two domains is constant.
- For a time domain record with duration T, the resolution in the frequency domain is proportional to 1/T.
- Perfect resolution in frequency requires infinite record length in time.
- Infinite bandwidth in frequency is needed to represent a time function exactly.

Tapered boxcar functions

$$W(t) = \frac{1}{2}\left[1+\cos\frac{\pi(t+T-T_1)}{T_1}\right], for -T<t<-T+T_1$$
$$= \frac{1}{2}\left[1+\cos\frac{\pi(t-T+T_1)}{T_1}\right], for T-T_1<t<T$$

Figure 6.3-10: Effects of tapering a boxcar function on the amplitude spectrum.



The side lobes for the tapered window are reduced, but the central peak is less sharp.

Similarly, band-pass filters are often tapered in the frequency domain.

Cross-correlation

$$C(L) = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x(t)f(t+L)dt$$

- $C(L)$, the cross-correlation of $x(t)$ and $f(t)$, measures the similarity between $f(t)$ and the later portions of $x(t)$ by shifting $f(t)$ by different lag times, L, and evaluating the integral of the product as a function of L.
- We often set T to an appropriate value, due to finite length of the data.

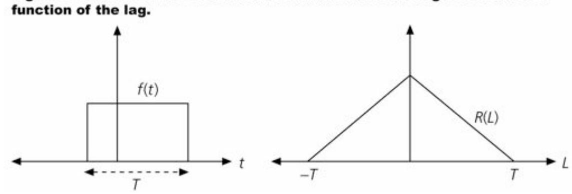
Auto-correlation

- A special case of the cross-correlation is the auto-correlation.

$$R(L) = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} f(t)f(t+L)dt$$

- The auto-correlation is maximum at zero lag, and is an even function of the lag.

Figure 6.3-12: The auto-correlation is maximum at zero lag and is an even function of the lag.



Auto-correlation and amplitude spectrum

$$R(L) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} f(t) f(t+L)\,dt$$

- Can be expanded using the inverse Fourier transform

$$\begin{aligned}
R(L) &= \lim_{T\to\infty} \frac{1}{2\pi T} \int_{-T/2}^{T/2} f(t) \left[\int_{-\infty}^{\infty} F(\varpi) e^{i\varpi(t+L)} d\varpi \right] dt \\
&= \lim_{T\to\infty} \frac{1}{2\pi T} \int_{-\infty}^{\infty} F(\varpi) e^{i\varpi L} \left[\int_{-T/2}^{T/2} f(t) e^{i\varpi t} dt \right] d\varpi \\
&= \lim_{T\to\infty} \frac{1}{2\pi T} \int_{-\infty}^{\infty} F(\varpi) F(-\varpi) e^{i\varpi L} d\varpi \\
&= \lim_{T\to\infty} \frac{1}{2\pi T} \int_{-\infty} |F(\varpi)|^2 e^{i\varpi L} d\varpi
\end{aligned}$$

Auto-correlation and amplitude spectrum

- If we define the power spectrum, a normalized version of the amplitude spectrum

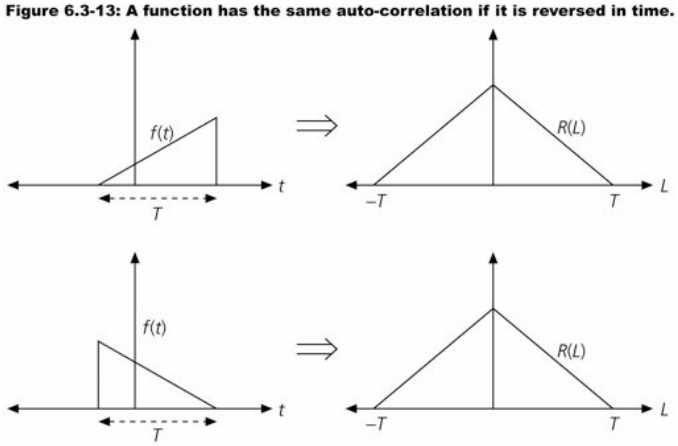
$$P(\omega) = \lim_{T\to\infty} \frac{1}{T} |F(\varpi)|^2$$

- Then the auto-correlation is the inverse Fourier transform of the power spectrum:

$$R(L) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |P(\varpi)| e^{i\varpi L} d\varpi$$

- As a result, the auto-correlation of a function contains information only about its amplitude spectrum, but not about its phase.

Figure 6.3-13: A function has the same auto-correlation if it is reversed in time.



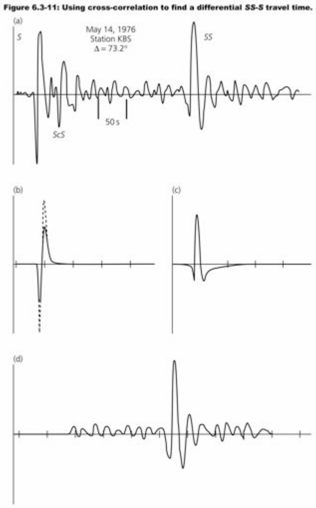
Cross-correlation and convolution

$$y(L) = \int_{-\infty}^{\infty} x(t) f(L-t)\,dt = x(t) * f(t)$$

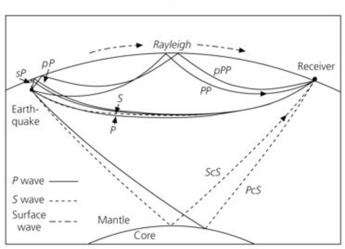
$$C(L) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) f(t+L)\,dt = x(t) \bullet g(t)$$

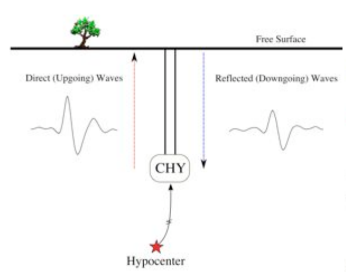
The cross-correlation is similar in nature to the convolution of two functions. Whereas convolution involves reversing a signal, then shifting it and multiplying by another signal, correlation only involves shifting it and multiplying (no reversing).

Figure 6.3-11: Using cross-correlation to find a differential *SS-S* travel time.

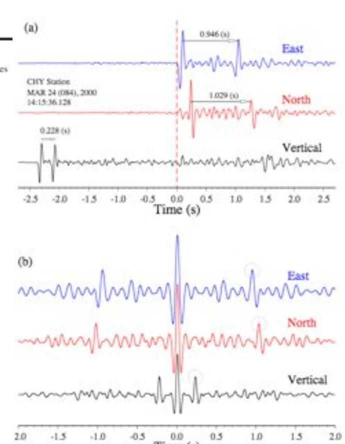


Application of the cross-correlation to determine the travel time difference between the direct *S* and reflected *SS* phases.





Temporal changes in the shallow crust from surface reflected P and S waves



Chao and Peng (GJI, 2009)

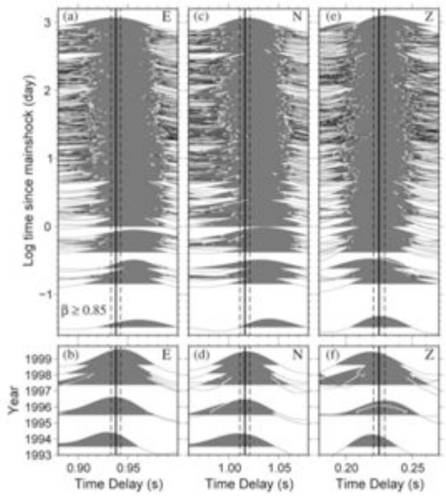
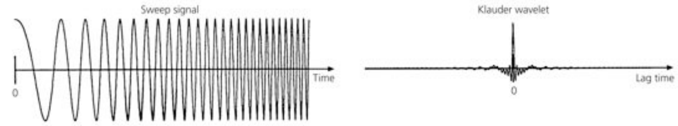


Figure 3.3-30: Auto-correlation of a Vibroseis sweep signal.

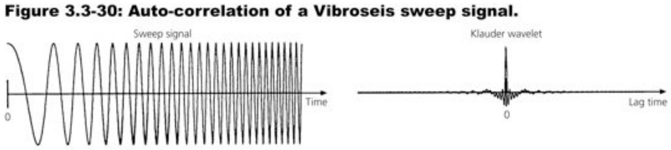


If $w(t)$ is a long signal, use cross-correlation.

The cross-correlation quantifies similarities between two time series $f(t)$ and $g(t)$:

$$c(L) = \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} f(t+L)g(t)dt$$

Figure 3.3-30: Auto-correlation of a Vibroseis sweep signal.



If $w(t)$ is a long signal, use cross-correlation.

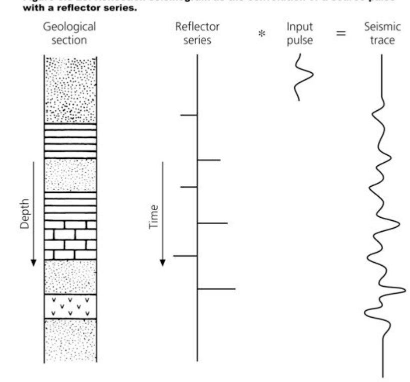
The cross-correlation quantifies similarities between two time series $f(t)$ and $g(t)$:

$$c(L) = \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} f(t+L)g(t)dt$$

For example, the cross-correlation of $w(t)$ with itself (called auto-correlation) is:

$$a(L) = \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} f(t+L)f(t)dt \qquad \text{(which is always maximum at zero lag)}$$

Figure 3.3-28: Reflection seismogram as the convolution of a source pulse with a reflector series.

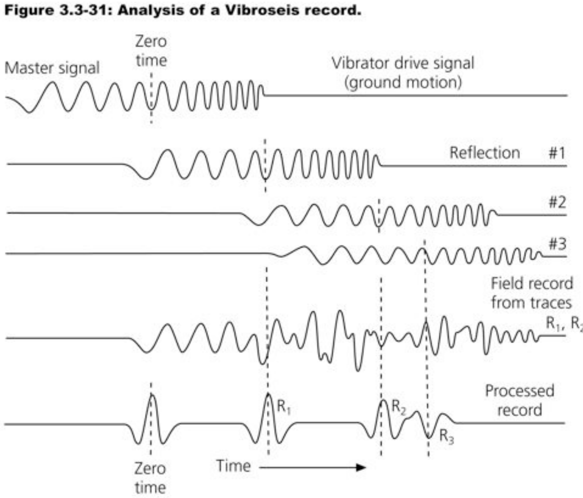


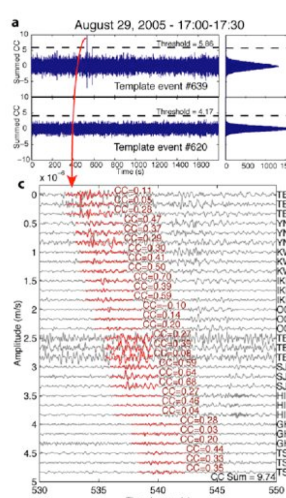
Convolution of $w(t)$ and $r(t)$:

$$s(t) = w(t) * r(t) \equiv \int_{-\infty}^{\infty} w(t-\tau)r(\tau)d\tau$$

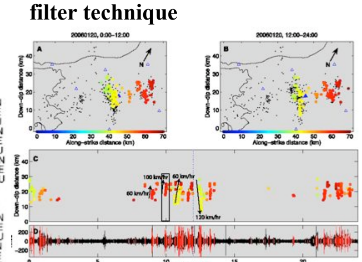
$$S(\omega) = W(\omega)R(\omega)$$

Figure 3.3-31: Analysis of a Vibroseis record.



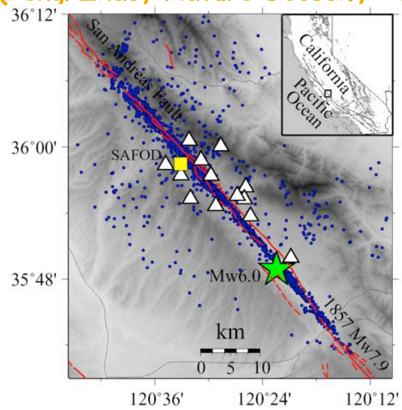


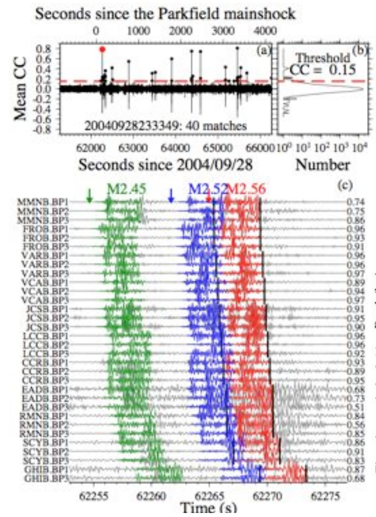
Search for low-frequency earthquakes within non-volcanic tremors by a waveform match-filter technique



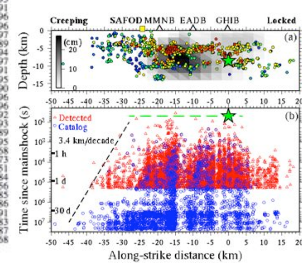
Shelly et al. (Nature, G3, 2007)

Early aftershocks of the 2004 Parkfield earthquake detected by a matched filter technique (Peng/Zhao, Nature Geosci., 2009)





We found 11 times more events in the first 2 days after the Parkfield mainshock!



Sampling of continuous data into discrete time series

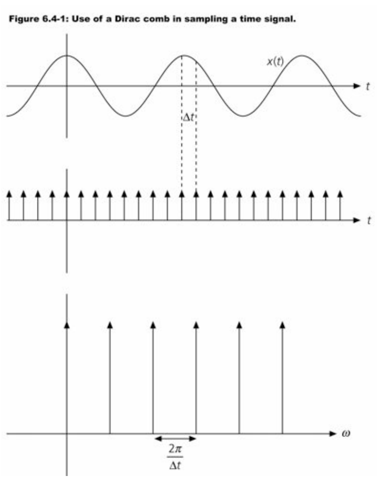


Figure 6.4-2: Effect of sampling a time signal on the frequency amplitude spectrum.

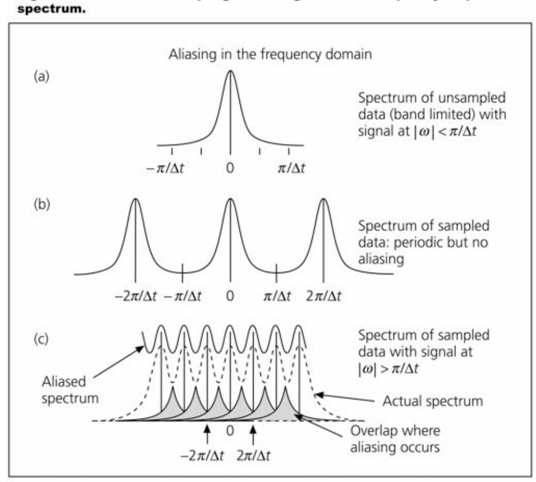
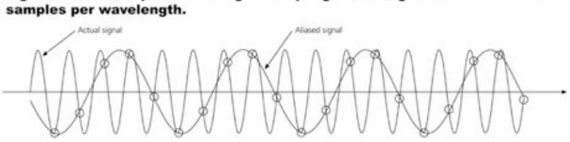


Figure 6.4-3: Example of aliasing in sampling a time signal at less than two samples per wavelength.

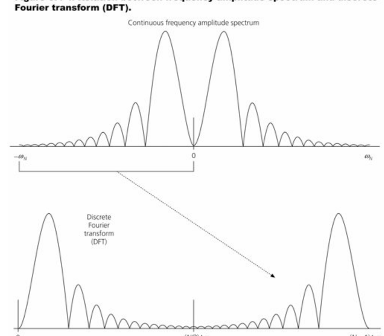


General rule:

1. At least two samples per wavelength are needed to reconstruct a sinusoid signals accurately.
2. For a sampling interval of Δt, the highest resolvable frequency is $f_N = 1/(2\Delta t)$, known as the Nyquist frequency.
3. Any frequencies higher than the Nyquist frequency are aliased into lower ones, when the data are sampled. This cannot be 'unaliased'.
4. Generally, seismic data are filtered with an analog anti-aliasing filter to remove frequencies above the Nyquist frequency before sampling to produce the digital seismogram.

Due to the periodicity of the discrete Fourier transform, the second half of the values of the frequency amplitude spectrum, at angular frequency greater than the Nyquist angular frequency $(N/2)\Delta\omega$, represents the negative frequencies.

Figure 6.4-4: Relation between frequency amplitude spectrum and discrete Fourier transform (DFT).



What we have learned today

- Linear systems
  – Basic models
  – Convolution and deconvolution modeling
  – Finite length signals
  – Correlation

Next class

- Seismometers and seismic network

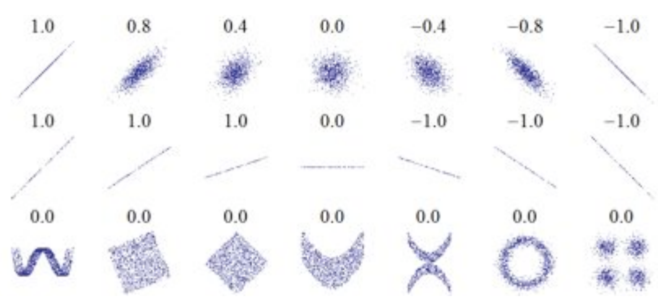


Figure 6.4-5: Example of a time domain convolution.