(6) If errors are uncorrelated (clocking error), then the covariance is ideally zero.
If errors are correlated (systematic bias in phase picking), then the covariance is non-zero.

(7) The data are inverted used the generalized inverse solution

$$m_j = \sum_i G^{-g}_{ji}\, d_i \qquad (31)$$

Covariance of the model parameter

$$\sigma_m^2 = \sigma_{m_{ji}}^2 = \lim_{K\to\infty} \frac{1}{K} \sum_{k=1}^{K} \left(m_j^{(k)} - \tilde{m}_j\right)\left(m_i^{(k)} - \tilde{m}_i\right)$$

$$= \lim_{K\to\infty} \frac{1}{K} \left[\sum_p G^{-g}_{jp}\left(d_p^{(k)} - \tilde{d}_p\right)\right]\left[\sum_s G^{-g}_{is}\left(d_s^{(k)} - \tilde{d}_s\right)\right]$$

$$= \sum_p G^{-g}_{jp} \sum_s G^{-g}_{is} \left[\lim_{K\to\infty} \frac{1}{K} \sum_{k=1}^{k} \left(d_p^{(k)} - \tilde{d}_p\right)\left(d_s^{(k)} - \tilde{d}_s\right)\right]$$

$$= \sum_p G^{-g}_{jp} \sum_s G^{-g}_{is}\, \sigma^2_{d_{ps}} \qquad (32)$$

Or in matrix form.

$$\sigma_m^2 = G^{-g}\, \sigma_d^2 \left(G^{-g}\right)^T \qquad (33)$$

If we assume that the data errors are uncorrelated and equal, then

$$\sigma_d^2 = \sigma^2 \delta_{ij} \qquad (34)$$

So the model variance - coviance matrix is

$$\sigma_m^2 = \sigma^2 \left(G^T G\right)^{-1} \qquad (35)$$

) Show Tabel 7.2.2. EQ location example with errors

9) In table 7.2.2, depth is less well ~~ed~~ constrained than the epicenter. Why?

10) The off-diagonal elements are non-zero, which means?

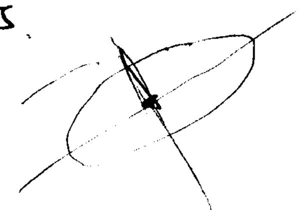
Ans: the uncertainties in the model parameter estimates are correlated

E.g. A trade-off between focal depth & origin time.

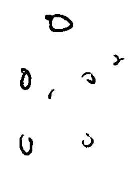
11) Error ellipse.

$$\begin{bmatrix} \sigma_{xx}^2 & \sigma_{xy}^2 \\ \sigma_{yx}^2 & \sigma_{yy}^2 \end{bmatrix}$$

diagonlize it by finding the eigenvalues $\lambda^{(1)}$ and $\lambda^{(2)}$, and associated eigenvectors.



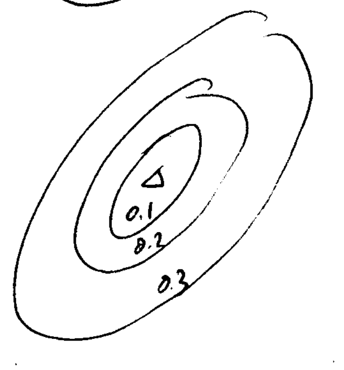
12) Behavior of the misfit function around its minimum



36

$$\epsilon = \sum_{i=1}^{n} [t_i - t_i^p]^2 .$$

~~Depending on how ϵ grow~~



If ϵ grows rapidly as we move away from the minimum point, we can ~~to~~ resolve the location better than a slow growth.

8

To quantify this
We define
$$\chi^2 = \sum_{i=1}^{n} \frac{[t_i - t_i^?]^2}{\sigma_i^2} \quad (37)$$

σ_i is the expected standard deviation of the ith residual due to random measurement error.

※ The expected value of χ^2 is $\approx$ the number of degrees of freedom ndf = number of observations − number of parameters

Percentage Points of the χ^2 Distribution

ndf	$\chi^2(95\%)$	$\chi^2(50\%)$	$\chi^2(5\%)$
5	1.15	4.35	11.07
10	3.94	9.34	18.31
20	10.85	19.34	31.41
50	34.76	49.33	67.50
100	77.93	99.33	124.34

The key is to get σ_i, sometimes the information is not available for a real experiment.

We could then estimate the standard deviation of the data from the misfit between the data and the best fitting model.

$$\sigma^2(m_{best}) = \frac{\sum_{i=1}^{n} [t_i - t_i^p(m_{best})]^2}{ndf} \qquad (38)$$

Where m_{best} is the best-fitting location.

Then eq (37) becomes.

$$\chi^2(m) = \frac{\sum_{i=1}^{n} [t_i - t_i^p(m)]^2}{\sigma^2} \qquad (39)$$

Note that $\chi^2(m_{best}) = ndf$ so that the χ^2-value at the best-fitting hypocenter is close to the 50% point in the χ^2 distribution. By contouring $\chi^2(m)$, we can then estimate the 95% confidence ellipse.