Total points: 100. Due 04/04/2011

1. (Computer. Question 4.8 in Peter Shearer's book: Introduction to Seismology, second edition) Consider MARMOD, a velocity versus depth model, which is typical of much of the oceanic crust (Table 1.1 below). Linear velocity gradients are assumed to exist at intermediate depths in the model; for example, the *P* velocity at 3.75 km is 6.9 km/s. Write a computer program to trace rays through this model and produce a *P*-wave $T(X)$ curve, using 100 value so the ray parameter *p* equally spaced between 0.1236 and 0.2217 s/km. You will find it help to use subroutine LAYERXT (provided in the following link), which gives *dx* and *dt* as a function of *p* for layers with linear velocity gradients. Your program will involve an outer loop over ray parameter and an inner loop over depth in the model. For each ray, set *x* and *t* to zero and then, starting with the surface layer and to the turning point. Multiply by two to obtain the total surface-to-surface values of $X(p)$ and $T(p)$. Now produce plots of:
   a. $T(X)$ plotted with a reduction velocity of 8 km/s;
   b. $X(p)$, and
   c. $\tau(p)$.
   (50 points)

Table 1.1 MARMOD, a Generic Marine Seismic Model

Depth (km)	α (km/s)	β (km/s)	ρ (g/cc)
0.0	4.50	2.40	2.0
1.5	6.8	3.75	2.8
6.0	7.00	3.85	2.9
6.5	8.00	4.60	3.1
10.0	8.10	4.70	3.1

Electronic version of this table:
http://geophysics.eas.gatech.edu/people/zpeng/Teaching/ObsSeis_2011/misc/hw4_table_11.txt

Link:
Fortran version of LAYERXT:
http://geophysics.eas.gatech.edu/zpeng/Teaching/ObsSeis_2011/ttime/layerxt.f
Matlab version of LAYERXT:
http://geophysics.eas.gatech.edu/zpeng/Teaching/ObsSeis_2011/ttime/layerxt.m

2. (Computer. Modified from question 5.2 in Peter Shearer's book: Introduction to Seismology, second edition) You are given *P*-wave arrival times for two earthquakes recorded by a 13-station seismic array. The station locations and times are listed in Table 2.1 (50 points).

a. Write a computer program that performs a grid search to find the best location for these events. Try every point in a 100 km by 100 km array ($x = 0$ to 100 km, $y = 0$ to 100 km). At each point, compute the range to each of the 13 stations. Convert these ranges to time by assuming the velocity is 6 km/s (this is a 2-D problem, don't worry about depth). Compute the variance (average sum of squares) of the residuals to each grid point (after finding the best-fitting origin time at the grid point; see below) and plot it out in a two-dimensional contour map for both events (15 points).

b. For each quake, list the best location, and origin time, and find the variance of the residuals at that point (10 points).

c. From your answers in (b), estimate the uncertainties of the individual station residuals σ^2 for each quake using the following equation:

$$\sigma^2(m_{best}) = \frac{\sum_{i=1}^{n}\left[t_i - t_i^p(m_{best})\right]}{ndf} \qquad (1)$$

where m_{best} is the best-fitting location, t_i is the observed travel time at each station, t^p is the predicted travel time, and ndf is the number of degrees of freedom, which is equal to the number of observations minus the number of free parameters (10 points).

d. For each quake, use (c) to computer χ^2 at each of the grid points from the following equation. What is χ^2 at the best fitting point in each case (5 points)?

$$\chi^2(m) = \frac{\sum_{i=1}^{n}\left[t_i - t_i^p(m)\right]}{\sigma^2} \qquad (2)$$

e. Identify those values of χ^2 that are within the 95% confidence ellipse from Table 2.2. For each quake, make a plot showing the station locations, the best quake location, and the points within the 95% confidence region (5 points).

f. Comment on what is the main reason of the difference in the 95% confidence regions for both earthquakes (5 points).

g. Note: Don't do a grid search for the origin time! Just assume an origin time of zero to start; the best-fitting origin time at each grid point will be the average of the residuals that you can calculate for that point. Then just subtract this time from all the residuals to obtain the final residuals at each point.

Table 2.1. *P*-Arrival Times for Two Earthquakes

Station	x (km)	y (km)	Quake 1 t1 (s)	Quake 2 t2 (s)
1	9.0	24.0	14.189	20.950
2	24.0	13.2	13.679	21.718
3	33.0	4.8	13.491	21.467
4	45.0	10.8	14.406	21.713
5	39.0	27.0	13.075	20.034

6	54.0	30.0	15.234	20.153
7	15.0	39.0	13.270	18.188
8	36.0	42.0	12.239	16.008
9	27.0	48.0	12.835	15.197
10	48.0	48.0	14.574	16.280
11	15.0	42.0	12.624	16.907
12	18.0	15.0	13.496	21.312
13	30.0	36.0	10.578	16.664

Table 2.2 Percentage Points of the χ^2 distribution

ndf	χ^2 (95%)	χ^2 (50%)	χ^2 (5%)
5	1.15	4.35	11.07
10	3.94	9.34	18.31
20	10.85	19.34	31.41
50	34.76	49.33	67.50
100	77.93	99.33	124.34

Note: the electronic version of the above two tables can be downloaded from
http://geophysics.eas.gatech.edu/people/zpeng/Teaching/ObsSeis_2011/misc/hw4_table_21.txt
http://geophysics.eas.gatech.edu/people/zpeng/Teaching/ObsSeis_2011/misc/hw4_table_22.txt

Note:
1. Your code can be written in any scientific languages (e.g., Fortran, C, Matlab). Please make sure that the code can be compiled under standard Linux machine. Please submit your code electronically to zpeng@gatech.edu, and submit a write-up that includes all the figures and answers to all questions.

2. The MatSAC package can be downloaded from the following link:
    http://geophysics.eas.gatech.edu/people/zpeng/Teaching/SAC_Tutorial/MatSAC.tar.gz
    Use fget_sac to read the SAC binary format data into Matlab.
    Alternatively, you can use sacdump program to extract data from SAC into two column ASCII output.