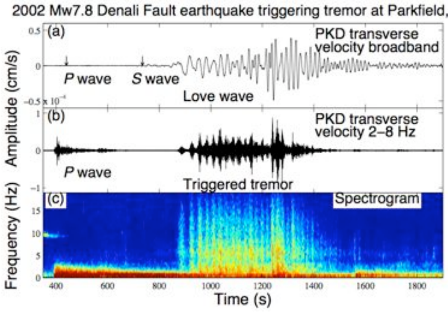
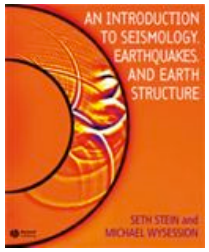
EAS 8803 - Observational Seismology

Lec#1: Introduction, Fourier Series

Dr. Zhigang Peng, Spring 2011

2002 Mw7.8 Denali Fault earthquake triggering tremor at Parkfield, CA





Today's Outline

- Course Introduction
  - Class logistics, requirements and policies
  - Class schedule
- Introduction to digital signal processing and its relation to seismological research
- Fourier series/transform

Reading: Stein and Wysession Chap. 6.1 – 6.2

Time and Place

- Lecture Time: M,W 3:05 pm – 4:25 pm
- Lecture Place: ES & T, L1116
- My office hour: M,W 1:30 pm – 2:30 pm

Class website:

http://geophysics.eas.gatech.edu/people/zpeng/Teaching/ObsSeis_2011/

Course Goals

- This is an advanced-level course designed to involve students into seismological research.
- The topics covered include digital signal processing, seismometers and seismic networks, basic and advanced seismic data processing tools, travel time and synthetic seismogram calculations, and modern topics in observational and computational seismology.

Grading

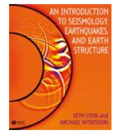
- 4 homework (40%)
- 3 weeks of paper reading and discussion (30%)
- Term paper project (30%)

Course outline – 1st half

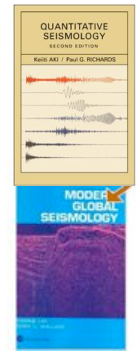
- **Digital Signal Processing**
  1. Fourier analysis
  2. Linear systems
  3. Discrete time series and transforms
- **Seismometers, Seismic Networks, and Data Centers**
  1. Historical development and the Earth's background noise
  2. The damped harmonic oscillator and instrument response
  3. Basic types of seismic sensors and digital recording devices
  4. Global and regional seismic networks and data management centers
  5. Optional Field Trip to Costa Rica
- **Observational Seismology**
  1. Basic data processing tools
  2. Data management
  3. Waveform stacking
  4. Array analysis

Course outline - 2nd half

- **Theoretical and Computational Seismology**
  1. Ray theory and travel time calculation
  2. Theoretical seismogram calculation
  3. Earthquake location and tomography
  4. Scattering
- **Current topics in observational and computational seismology**
  1. Ambient noise tomography and seismic interferometry
  2. Waveform back projection for imaging earthquake ruptures
  3. Spectral-element methods (SEM) and full-waveform tomography



Text Book



Required:

S. Stein and M. Wysession, An Introduction to Seismology, Earthquakes, and Earth Structure Blackwell Publishing.

Recommended:

K. Aki and P.G. Richards, Quantitative Seismol W.H. Freeman and Co.

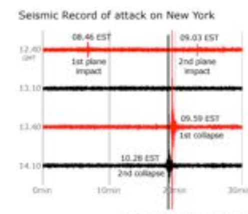
T. Lay and T.C. Wallace, Modern Global Seismology, Academic Press.

Additional material will be either handed out in class or made available on the course website.

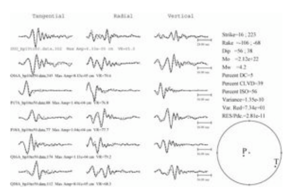
Why seismology is interesting?

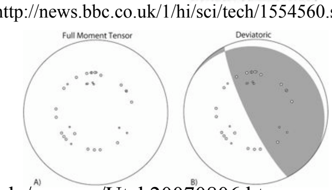
- Seismology (wikipedia): is the scientific study of earthquakes and the movement of waves through the Earth.
- Earthquakes, and other earth movements, produce different types of seismic waves.
- These waves travel through rock, and provide an effective way to "see" events and structures deep in the Earth.
- What are other types of events (not earthquakes) generating seismic signals?

Man-made signals

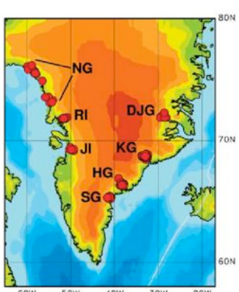


http://news.bbc.co.uk/1/hi/sci/tech/1554560.stm



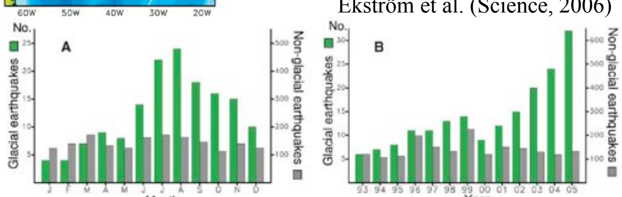


http://seismo.berkeley.edu/~peggy/Utah20070806.htm



Other natural events that generate seismic waves: Glacial earthquakes in Greenland. Evidence of global warming?

Ekström et al. (Science, 2006)



Signal and Noise

- What is the definition of signal and noise?

•"We shall introduce the concepts of signal and noise. We define the signal as the desired part of the data and the noise as the unwanted part".

•"Our definition of signal and noise is subjective in the sense that a given part of the data is "signal" for those who know how to analyze and interpret the data, but it is "noise" for those who do not".

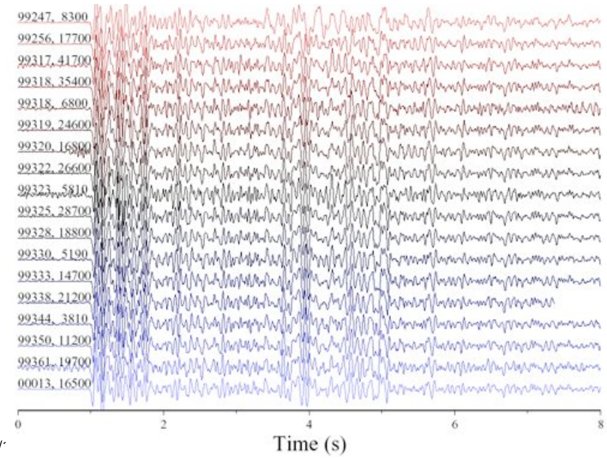
Aki and Richards, Quantitative Seismology, 1980

Signal and Noise

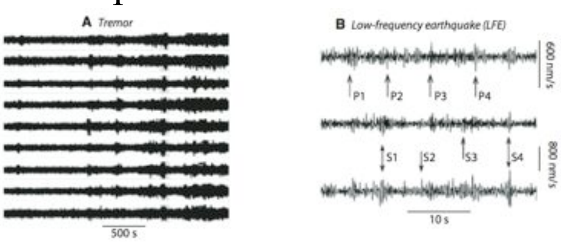
- "For example, for many years the times of the first arrivals of P- and S-waves were the only signals conveyed by an earthquake, and the rest of the seismograms, such as surface waves and coda waves, had to be considered as useless until appropriate methods of interpretations were found.
- Thus, through the application of a new technique to old data, an analyst (seismologists) can experience a moment of discovery as joyful as a data gather does using a new observational device."

Can you think of any examples of noise turning into signal in the field of seismology?

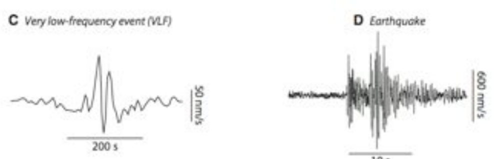
Example 1: coda waves



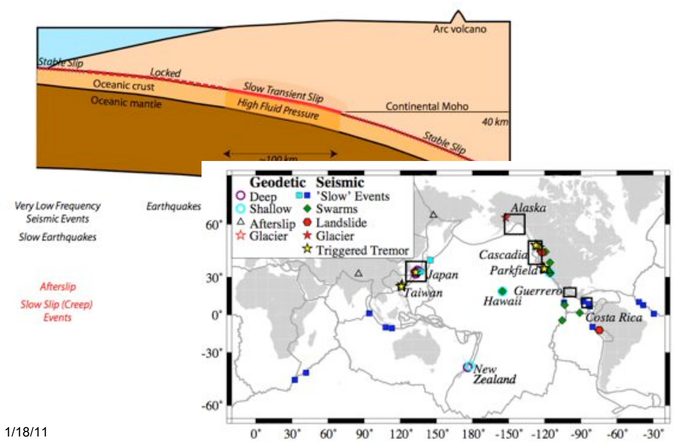
Example 2: Non-volcanic tremors



Gomberg et al., GSA (2010)



Global observations of slow-slip phenomena (Peng and Gomberg, NGEO, 2010)



Example 3: Ambient noise tomography

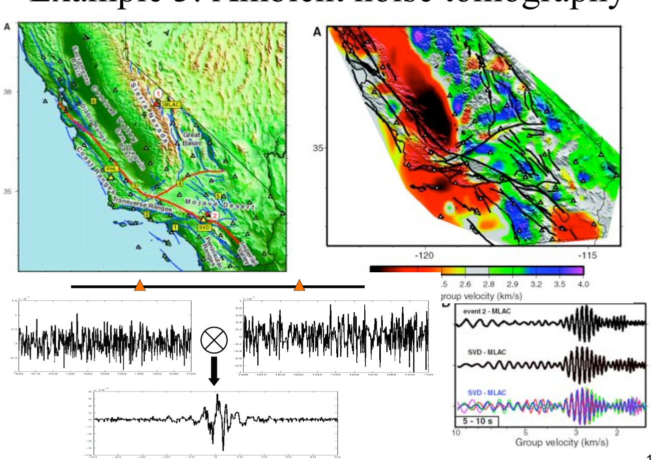
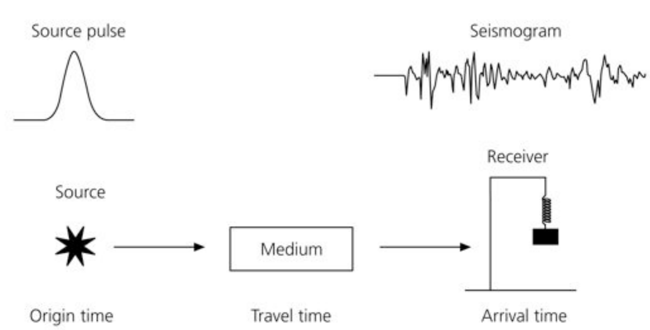


Figure 1.1-1: Schematic geometry of a seismic experiment.



Filtering: applying some operations that modifies the signal.
The Earth is a "low-pass filter".
A seismometer is a "band-pass filter".

Relation between seismology and signal processing

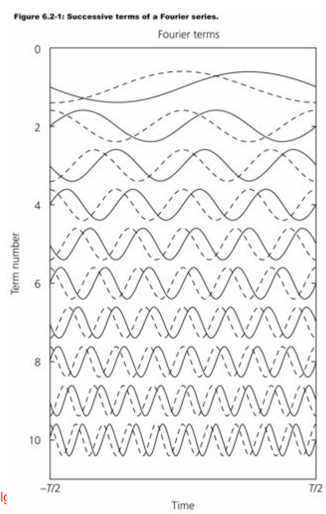
- Seismology uses various techniques to study the displacement (or velocity, acceleration) as a function of position and time associated with elastic waves, and to draw conclusions about the seismic sources and the earth.
- A major task is seismology is to separate the source, path and site effects in order to study each of them in details.

Relation between seismology and signal processing

- Signal processing (or time series analysis) considers functions of space or time in general terms with regard to the specific physics involved.
- Hence, many wave propagation subjects, including seismology, radar, sonar, and optics, can be treated in similar ways via signal processing technique.

Fourier analysis

- Any time series can be decomposed into the sum or integral of harmonic waves of different frequencies.
- Harmonic waves: a sinusoid with a single frequency.



Fourier Series

- To find the coefficients a_n and b_n:

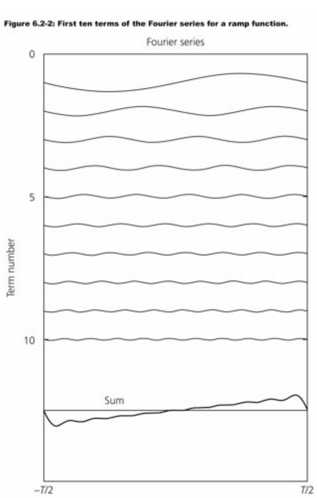
$$\int_{-T/2}^{T/2}\cos\left(\frac{2k\pi t}{T}\right)f(t)dt=\int_{-T/2}^{T/2}\cos\left(\frac{2k\pi t}{T}\right)\left[a_0+\sum_{n=1}^{\infty}a_n\cos\left(\frac{2n\pi t}{T}\right)+\sum_{n=1}^{\infty}b_n\sin\left(\frac{2n\pi t}{T}\right)\right]dt$$

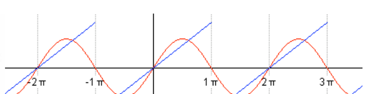
- The only nonzero term is $\cos(2\pi kt/T)$, so

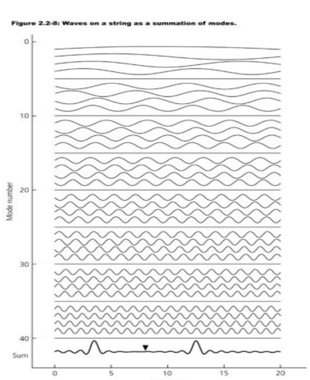
$$\int_{-T/2}^{T/2}\cos\left(\frac{2k\pi t}{T}\right)f(t)dt=\int_{-T/2}^{T/2}\cos^2\left(\frac{2k\pi t}{T}\right)dt=\frac{T}{2}a_k(1+\delta_{k0}),$$

$$a_k=\frac{2-\delta_{k0}}{T}\int_{-T/2}^{T/2}\cos\left(\frac{2k\pi t}{T}\right)f(t)dt \qquad a_0=\frac{1}{T}\int_{-T/2}^{T/2}f(t)dt$$

$$b_k=\frac{2}{T}\int_{-T/2}^{T/2}\sin\left(\frac{2k\pi t}{T}\right)f(t)dt$$







Complex Fourier Series

- The Fourier series can be written in a simpler form by expanding the sine and cosine functions into complex exponentials, so that the Fourier series becomes

$$f(t)=F_0+\sum_{n=1}^{\infty}[F_n e^{iw_n t}+F_{-n}e^{-iw_n t}]$$

- The negative exponentials can be written as

$$\sum_{n=1}^{\infty}F_{-n}e^{-iw_n t}=\sum_{n=-1}^{-\infty}F_n e^{iw_n t}$$

- So the Fourier series can be written in complex number form as:

$$f(t)=\sum_{n=-\infty}^{\infty}F_n e^{iw_n t} \qquad F_n=\frac{1}{T}\int_{-T/2}^{T/2}e^{-i\varpi_n t}f(t)dt$$

From Fourier Series to Fourier Transforms

- Fourier Series: a time series expressed in terms of a sum over discrete angular frequencies $\varpi_n = 2n\pi / T$
- Fourier Transforms: a time series expressed as an integral of a continuous range of angular frequencies.
- Fourier Transforms are used in most seismological application, because we regard the waves as continuous functions of angular frequencies

From Fourier Series to Fourier Transforms

- Rewrite $f(t) = \sum_{n=-\infty}^{\infty} F_n e^{i\varpi_n t} = \sum_{n=-\infty}^{\infty} F_n e^{i\varpi_n t} \Delta n$
- where $\Delta n = 1$.
- Since $\Delta\varpi = 2\pi / T\Delta n$
- so that $\Delta n = (T / 2\pi)\Delta\varpi$
- and $f(t) = \sum_{n=-\infty}^{\infty} F_n e^{i\varpi_n t} (T / 2\pi)\Delta\varpi$
- If we let the period T go to infinity,

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\varpi) e^{i\varpi t} d\varpi \qquad F(\varpi) = \int_{-\infty}^{\infty} e^{-i\varpi t} f(t) dt$$

Fourier Transforms

- If $f(t)$ is a seismogram that has the dimensions of displacement, its Fourier transform $F(\omega)$ has the dimensions of displacement multiplied by time (from the *dt* term).
- The Fourier transform can be written in terms of two real-valued functions of ω:

$$F(\varpi) = |F(\varpi)| e^{i\phi(\omega)}$$ Amplitude spectrum

$$|F(\varpi)| = [F(\varpi)F^*(\varpi)]^{1/2} = [\mathrm{Re}^2(F(\varpi)) + \mathrm{Im}^2(F(\varpi))]^{1/2}$$

$$\phi(\varpi) = \tan^{-1}(\mathrm{Im}(F(\varpi)) / \mathrm{Re}(F(\varpi)))$$ Phase spectrum

Figure 6.2-3: Amplitude spectra for the body and surface wave segments from a large earthquake.

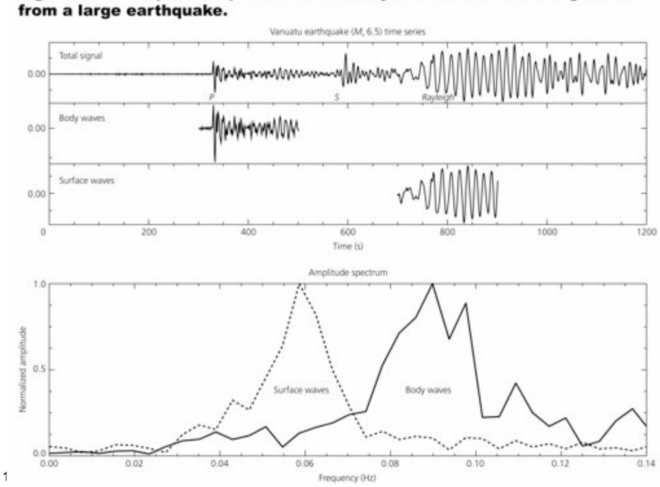
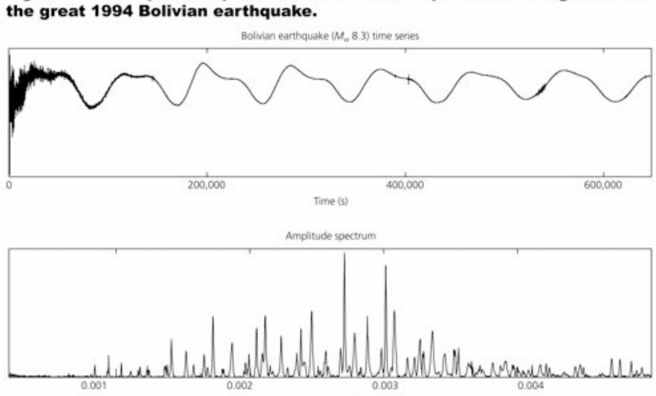


Figure 6.2-4: Amplitude spectra of a vertical-component seismogram from the great 1994 Bolivian earthquake.



Time and frequency domain

- Time domain: time series $f(t)$
- Frequency domain: $F(\omega)$
- Can you think of another pairs of representation in different domain?

- Spatial domain: Distance (*d*, or wavelength)
- Wavenumber domain: wavenumber (*k*)

Properties of Fourier Transform (I)

- The Fourier transform is linear: if $F(\omega)$ and $G(\omega)$ are the transforms of $f(t)$ and $g(t)$, then $(aF(\omega) + bG(\omega))$ is the transform of $(af(t) + bg(t))$.
- The Fourier transform of a purely real time function has the symmetry

$$F(-\varpi) = F^*(\varpi)$$

Properties of Fourier Transform (II)

- The Fourier transform of a time series shifted in time is found by changing the phase of the transform: if the transform of $f(t)$ is $F(\omega)$, the transform of $f(t-a)$ is $e^{-i\varpi a}F(\varpi)$
- Similarly, shifting of a Fourier transform in frequency domain causes a phase change in the corresponding time series: the inverse transform of $F(\omega-a)$ is $e^{iat}f(t)$

Properties of Fourier Transform (III)

- The Fourier transform of the derivative of a time function is found by multiplication: $(i\omega)F(\omega)$ is the transform of $df(t)/dt$.
- This makes differentiation easy in the frequency domain, and make it easy to solve differential equations.
- The total energy in a Fourier transform is the same as that in the time series (Parserval theorem):

$$\int_{-\infty}^{\infty}|f(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|F(\varpi)|^2\,d\varpi$$

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Next time

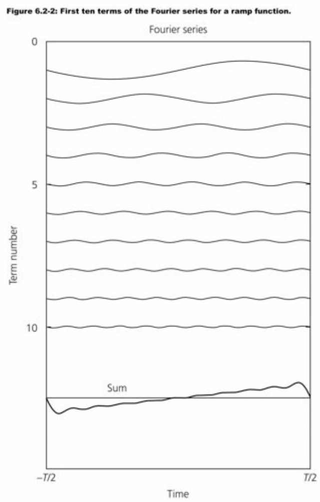
- Fourier transforms/Linear systems

Reading: Stein and Wysession Chap. 6.1 – 6.2

"I cannot help feeling that seismology will stay in the place at the center of solid earth science for many, many years to come…

The joy of being a seismologist comes to you, when you find something new about the earth's interior from the observation of seismic waves obtained on the surface, and realize that you did it without penetrating the earth or touching or examining it directly."

Keiiti Aki, presidential address to the Seismological Society of America, 1980



For a ramp function $f(t) = t/T$, performing the integration gives

$$a_k = 0 \quad b_k = (-1)^{k+1}/k\pi$$

The cosine terms are zero, why?

Fourier Series

- Decomposition of a signal with a finite duration

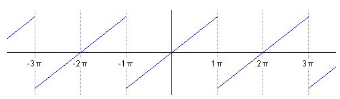
$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2n\pi t}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2n\pi t}{T}\right)$$

- The sine and cosine Fourier terms are a set of *orthogonal* functions.

$$\int_{-T/2}^{T/2} \sin\left(\frac{2m\pi t}{T}\right)\sin\left(\frac{2n\pi t}{T}\right)dt = \frac{T}{2}\delta_{mn}(1-\delta_{m0}),$$
$$\int_{-T/2}^{T/2} \cos\left(\frac{2m\pi t}{T}\right)\cos\left(\frac{2n\pi t}{T}\right)dt = \frac{T}{2}\delta_{mn}(1+\delta_{m0}),$$
$$\int_{-T/2}^{T/2} \cos\left(\frac{2m\pi t}{T}\right)\sin\left(\frac{2n\pi t}{T}\right)dt = 0 \quad \text{For all } m, n.$$

Simple Fourier series

$$f(x) = x, \quad \text{for} \quad -\pi < x < \pi$$
$$f(x+2\pi) = f(x), \quad \text{for} \quad -\infty < x < \infty$$



$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x\cos(nx)\,dx = 0.$$

$$\begin{aligned} b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx \\ &= \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin(nx)\,dx \\ &= \frac{2}{\pi}\int_{0}^{\pi} x\sin(nx)\,dx \\ &= \frac{2}{\pi}\left(\left[-\frac{x\cos(nx)}{n}\right]_0^{\pi} + \left[\frac{\sin(nx)}{n^2}\right]_0^{\pi}\right) \\ &= 2\frac{(-1)^{n+1}}{n}. \end{aligned}$$

$$\begin{aligned} f(x) &= \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n\cos(nx) + b_n\sin(nx)\right] \\ &= 2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin(nx), \quad \text{for} \quad -\infty < x < \infty. \end{aligned}$$

http://en.wikipedia.org/wiki/Fourier_series