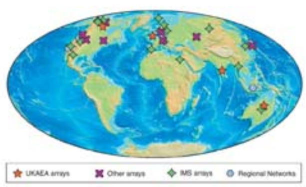
EAS 8803 - Seismology II
Lec#13: Array Analysis

- Dr. Zhigang Peng, Spring 2008



Last Time

- Stacking in exploration geophysics
- Stacking to obtain reliable deep Earth structure
- Stacking to estimate seismic source properties

This Time

- Data management and basic data processing tools
- Systematic and random errors
- Waveform stacking
- Array analysis

Array Analysis

- Introduction of array
- Basic array processing techniques
- Example of array processing techniques for Earth structures
- Example of array processing techniques for earthquake source properties

Definition

- Seismic array: many uniform seismometers in a well-defined, closely-spaced configuration (*Rost and Thomas*, Rev. Geophys., 2002).
- *Rost and Garnero* (EOS, 2004) gave the following criteria for seismic array:
  - Three or more seismometers
  - An aperture of more than 1 and and less than a few hundred kms
  - Uniform instrumentation and recording
  - A means of analysis of the data as an ensemble
  - A common time signal.

Definition

- Array processing techniques: methods of using the abilities of seismic arrays to measure the vector velocities of an incident wavefront, i.e., slowness and back azimuth.
- Difference between global and regional seismic network: more focused in the purpose, more strict in their configuration, and different analysis tools.

Figure 6.6-16: Station map of the Federation of Digital Broad-Band Seismographic Networks (FDSN).

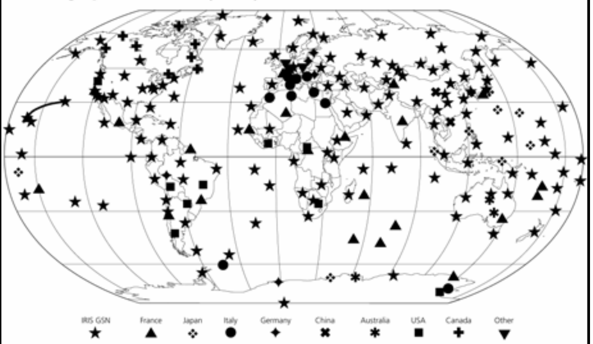


Figure 6.6-18: Map of regional network seismometers in the continental USA.

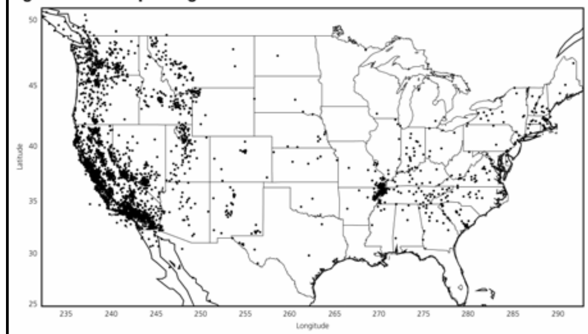
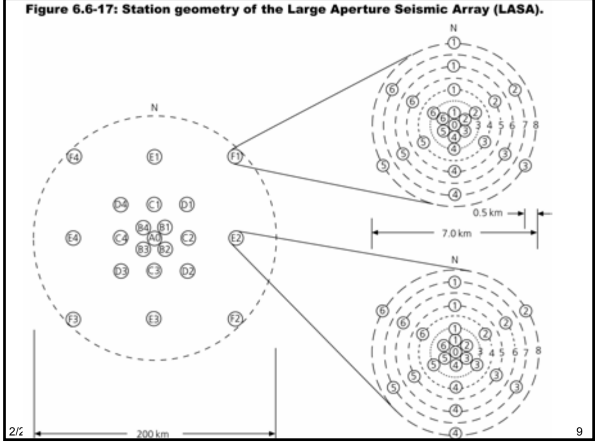
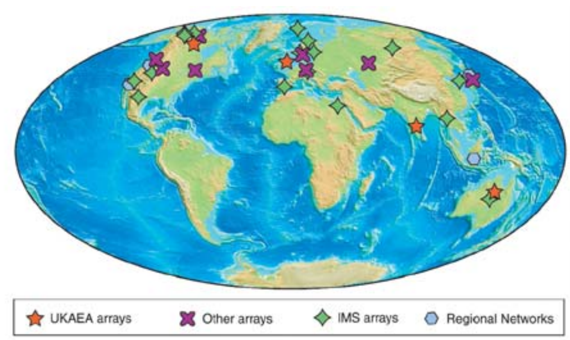


Figure 6.6-17: Station geometry of the Large Aperture Seismic Array (LASA).





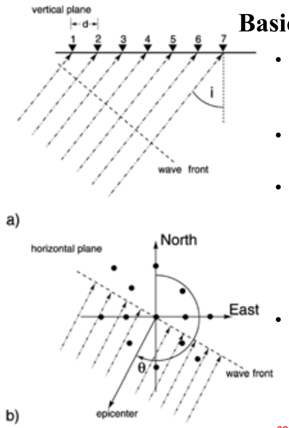
Why should we use arrays?

- This information can be used to distinguish between different seismic phases, separate waves from different seismic events and improve the signal-to-noise ratio by stacking with respect to the varying slowness of different phases.
- The vector velocity information of scattered or reflected phases can be used to determine the region of the Earth from whence the seismic energy comes and with what structures it interacted.

Why should we use arrays?

- Therefore seismic arrays are perfectly suited to study the fine-scale structure and spatio-temporal variations of the material properties of the Earth’s interior.
- Array analysis can also be used to better quantify the seismic source mechanisms (e.g., rupture duration, velocity, areas, etc), and forensic seismology (Nuke detection, terrorist attacks, etc).

Basics of Array Analysis (AA)



- Most array analysis methods assume a plane wave arriving at the array.
- This is a good approximation for teleseismic events.
- The propagation of an elastic waves can be described by two parameters:
  - Vertical incident angle i,
  - Back azimuth θ.
- In practice, we often use the inverse of the apparent velocity of the wavefront $1/V_{app}$

$$u = \frac{1}{V_{app}} = \frac{\sin i}{V_0}$$

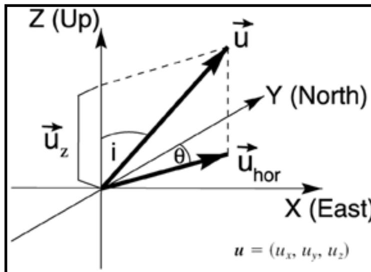


Figure 2. Components of the slo nents incident angle i and back az are marked. The slowness vector is front.

$$\boldsymbol{u} = (u_x, u_y, u_z)$$

$$= \left(\frac{\sin\theta}{v_{app}}, \frac{\cos\theta}{v_{app}}, \frac{1}{v_{app}\tan i}\right)$$

$$= u_{hor}\left(\sin\theta, \cos\theta, \frac{1}{\tan i}\right)$$

$$= \frac{1}{v_0}(\sin i \sin\theta, \sin i \cos\theta, \cos i). \quad (2)$$

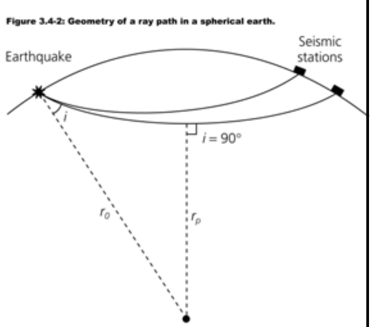
A the surface:

$$p = \frac{r_0}{v_0 \sin i} = r_0 u$$

At the bottoming depth, $r = r_p$, and

$$p = \frac{r_p}{v_p}$$

Figure 3.4-2: Geometry of a ray path in a spherical earth.



The slowness $\boldsymbol{u}$ is a way to identify different phases traveling through the Earth's interior as it is unique to a given phase in a one-dimensional Earth.

Beam forming

- An important use of seismic arrays is the separation of coherent signals and noise. The basic method to separate coherent and incoherent parts of the recorded signal is array beam forming.
- Beam forming uses the differential travel times of the plane wave front due to a specific slowness and back azimuth to individual array stations.
- If the single-station recordings are appropriately shifted in time for a certain back azimuth and slowness, all signals with the matching back azimuth and slowness will sum constructively.

Station Geometry

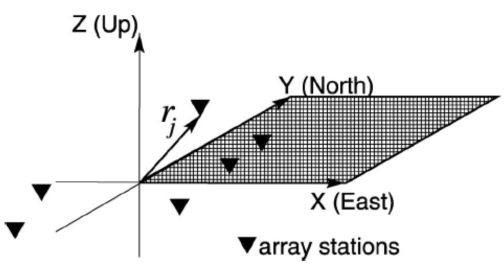


Figure 3. The definition of the sensor position vectors $\mathbf{r}_j$. The center of the array is assumed to be in the center of the Cartesian coordinate system.

Delay and Sum

The incident wavefield at the array center

$$x_{center}(t) = f(t) + n_i(t).$$

Station i with the location r_i records the time series:

$$x_i(t) = f(t - \mathbf{r}_i \cdot \mathbf{u}_{hor}) + n_i(t)$$

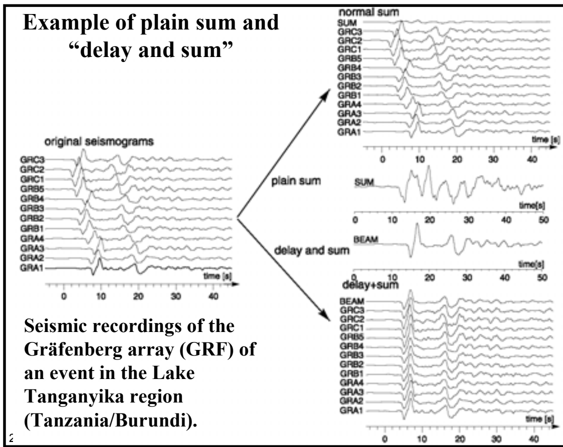
with r_i representing the location vector of station i and u_{hor} representing the horizontal slowness vector.

$$\tilde{x}_i(t) = x_i(t + \mathbf{r}_i \cdot \mathbf{u}_{hor}) = f(t) + n_i(t + \mathbf{r}_i \cdot \mathbf{u}_{hor}).$$

The "delay and sum" beam trace for an array with M components is then computed by

$$b(t) = \frac{1}{M}\sum_{i=1}^{M}\tilde{x}_i(t) = f(t) + \frac{1}{M}\sum_{i=1}^{M} n_i(t + \mathbf{r}_i \cdot \mathbf{u}_{hor}).$$

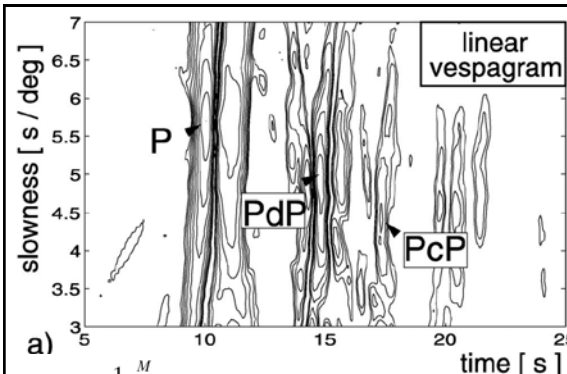
Example of plain sum and "delay and sum"



Seismic recordings of the Gräfenberg array (GRF) of an event in the Lake Tanganyika region (Tanzania/Burundi).

Vespa Process–Slant Stacks

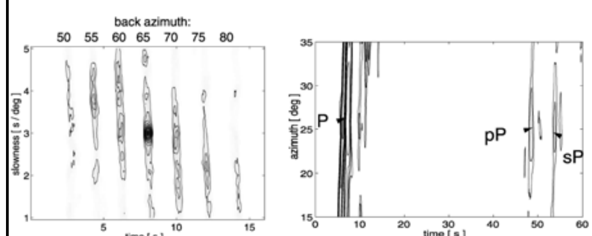
- The beam forming method enhances the amplitude of a signal with a given slowness ***u***.
- To determine the unknown horizontal slowness or the back azimuth of an arriving signal, the so-called vespa process (velocity spectral analysis [*Davies et al.*, 1971]) can be used.
- The vespa in its original form [*Davies et al.*, 1971] estimates the seismic energy arriving at the array for a given back azimuth and different horizontal slownesses ***u***.
- Alternatively, the vespa process can be used for a fixed slowness and varying back azimuths.
- The result of the vespa process is displayed as a vespagram, a diagram of the energy content (amplitudes) of the incoming signals as a function of slowness or back azimuth and time.



$$v_u(t) = \frac{1}{M}\sum_{i=1}^{M} x_i(t - t_{u,i}),$$

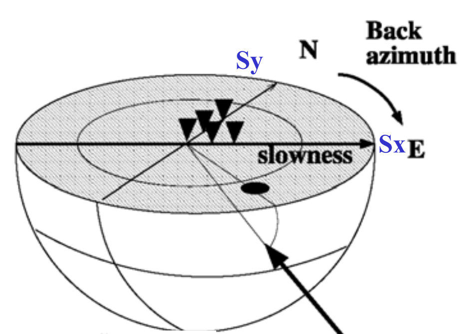
$$g = \frac{\pi\ 6371\ \text{km}}{180°} \cong 111.91\ \text{km/°}.$$

A wrong back azimuth (slowness) used for the computation may produce misleading slowness (back azimuth) measurements.



Frequency-wave number analysis

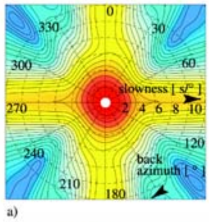
- In contrast to the array methods previously introduced, the frequency-wave number analysis (fk analysis) can measure the complete slowness vector (i.e., back azimuth θ and horizontal slowness u) simultaneously.
- A grid search for all u and θ combinations can be performed to find the best parameter combination, producing the highest amplitudes of the summed signal.

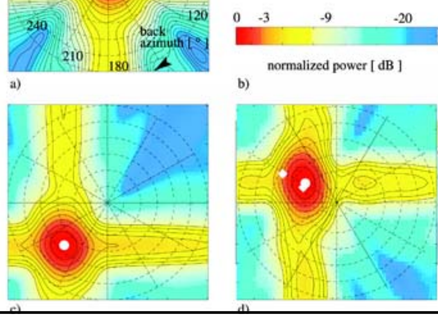


$$\mathbf{k} = (k_x, k_y) = \omega \cdot \mathbf{u} = \frac{\omega}{v_0}(\cos\theta, \sin\theta)$$

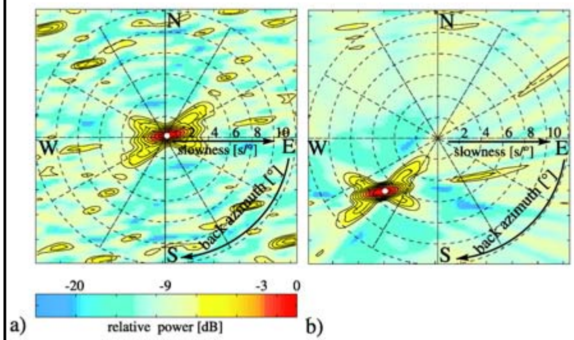
$$v_{app} = 1/u_{hor} = \frac{1}{\sqrt{s_x^2 + s_y^2}} \qquad \theta = \tan^{-1}\left(\frac{s_x}{s_y}\right)$$

Array response function (ARF) of the small-aperture Yellowknife array (YKA) in northern Canada.



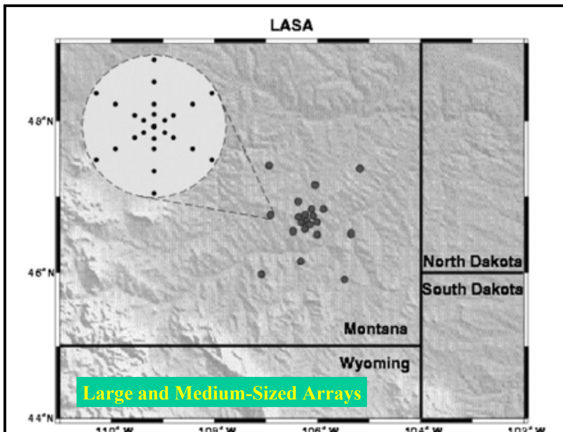


Array response function of GRF.



Array design principle

- Depending on the application of the array (detection, frequency of interest), their geometries vary significantly.
- Design principle:
  - The ARF should have a sharp main lobe, ideally a delta pulse with a strong suppression of the energy next to the main lobe.
  - The sidelobes due to spatial aliasing should not be within the wave number window of interest.
  - The aperture of the array affects the sharpness of the main lobe, i.e., the resolution of the array.
  - The interstation spacing defines the position of the sidelobes in the ARF and the largest resolvable wave number; that is, the smaller the interstation spacing, the larger the wavelength of a resolvable seismic phase will be.



Small Arrays

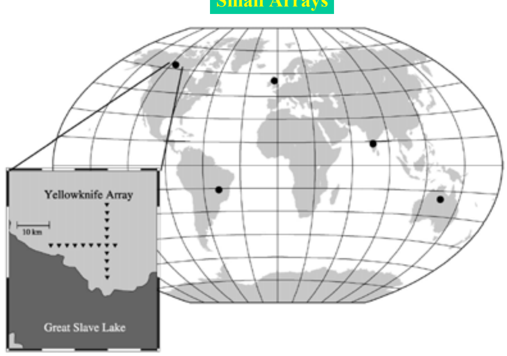
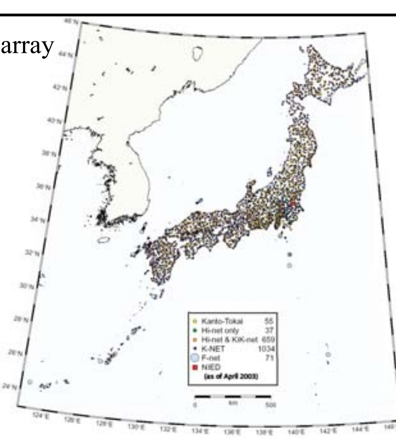
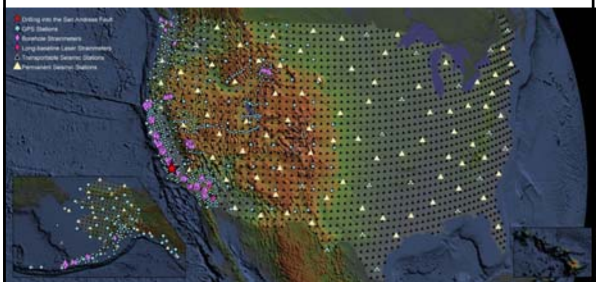


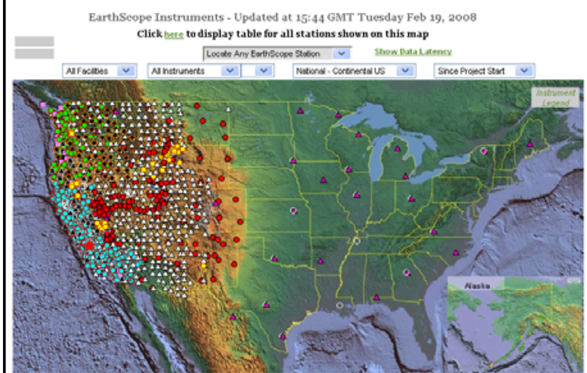
Figure 21. Location of the U.K. Atomic Energy Administration type of array. The arrays are EKA (Eskdalemuir, Scotland), GBA (Gauribidanur, India), WRA (Warramunga, Australia), BDF (Brasilia), and YKA (Yellowknife, Canada). The inset shows the configuration of the Canadian Yellowknife array.

Networks as an array



US Array





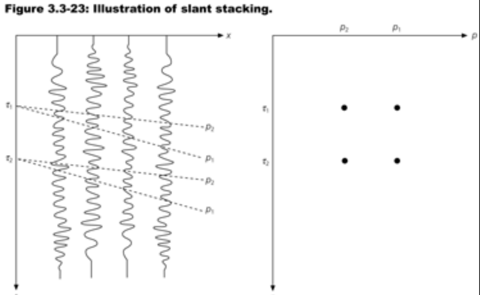
This Time

- Data management and basic data processing tools
- Systematic and random errors
- Waveform stacking
- Array analysis

No class next week

- Two homework assignments.
- Further reading lists:
  - Rost, S., and C. Thomas (2002), Array seismology: Methods and applications, Rev. Geophys., 40(3), 1008, doi:10.1029/2000RG000100.
  - http://geophysics.eas.gatech.edu/internal/papers/2002/Rost/Rost_Thomas_RG_2002.pdf
  - S. Rost and E.J. Garnero (2004), Array seismology advances Earth interior research, EOS, 85, 301, 305-306.
  - http://geophysics.eas.gatech.edu/internal/papers/2004/Rost/Rost_Garnero_EOS_2004.pdf

Figure 3.3-23: Illustration of slant stacking.



$$\bar{u}(\tau, p) = \int_{-\infty}^{\infty} u(x, \tau + px)dx$$

This integral (*slant stack*) maps all the data along each slanted line in (x, t) to a point in (τ, p).