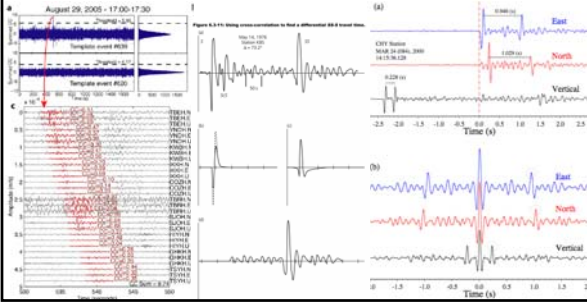


EAS 8803 - Seismology II

Lec#4: Linear Systems (cont.)

Dr. Zhigang Peng, Spring 2008



Last Time

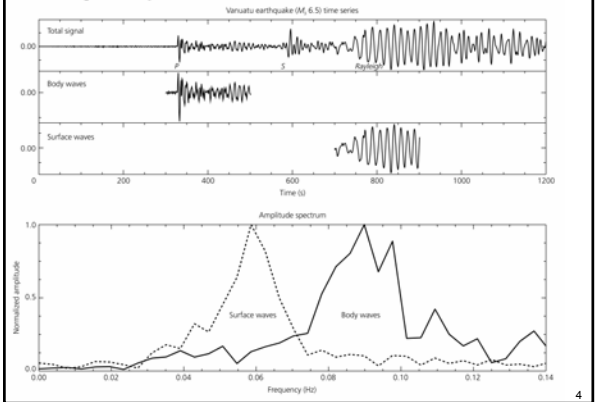
- Linear systems
 - Basic models
 - Convolution and deconvolution modeling
 - Finite length signals
 - Correlation

This Time

- Linear systems
 - Basic models
 - Convolution and deconvolution modeling
 - Finite length signals
 - Correlation

Reading: Stein and Wysession Chap. 6.3-6.4

Figure 6.2-3: Amplitude spectra for the body and surface wave segments from a large earthquake.



Finite length signals

- Consider a window function $b(t)$. Its effect on the data $f(t)$ is represented by multiplying $f(t)$ by $b(t)$.

$$G(\omega) = \int_{-\infty}^{\infty} b(t) f(t) e^{-i\omega t} dt$$

$$G(\omega) = \int_{-\infty}^{\infty} \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} B(\omega') e^{i\omega' t} d\omega' \right] \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega'') e^{i\omega'' t} d\omega'' \right] e^{-i\omega t} dt$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} B(\omega') \left[\int_{-\infty}^{\infty} F(\omega'') \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i(\omega + \omega' - \omega'') t} dt \right] d\omega'' \right] d\omega'$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} B(\omega') \left[\int_{-\infty}^{\infty} F(\omega'') \delta(\omega - \omega' - \omega'') d\omega'' \right] d\omega'$$

Finite length signals

- Using the sifting properties, we obtain

$$G(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} B(\omega') \left[\int_{-\infty}^{\infty} F(\omega'') \delta(\omega - \omega' - \omega'') d\omega'' \right] d\omega'$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} B(\omega') F(\omega - \omega') d\omega' = \frac{1}{2\pi} B(\omega) * F(\omega)$$

- Hence, the effect of multiplying a time series by a window function is that the spectrum of the time series is convolved with the spectrum of the window function.
- This is what is expected!

Effects of a boxcar window function

$$b(t) = 1 \text{ for } -T < t < T, \\ = 0, \text{ otherwise.}$$

Its Fourier transform is:

$$B(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} dt = \frac{e^{-i\omega T}}{-i\omega} \Big|_{-T}^T = \frac{2 \sin \omega T}{\omega} = \frac{2T \sin \omega T}{\omega T}$$

Figure 6.3-8: A boxcar function in the time and frequency domains.

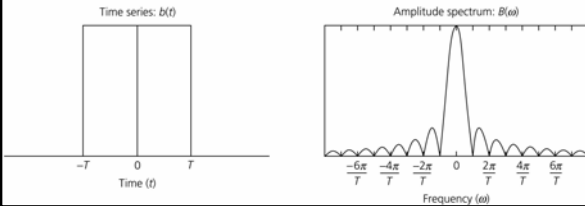
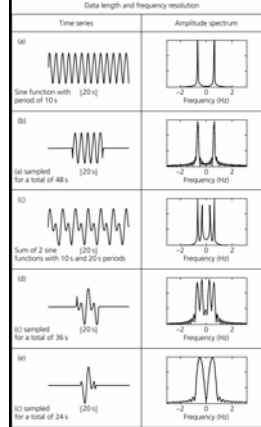


Figure 6.3-9: Effects of windowing time signals on the amplitude spectrum.



$f(t)$ is a sine wave.
What's the effect?

Taking a finite length of record "smears" the delta functions of the infinite length record's spectrum into broader peaks with side lobes.

Input signals contains different frequencies

The frequency resolution, the minimum separation in frequency for which two peaks can be resolved, is proportional to the reciprocal of the total length.

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"Uncertainty principle" in time and frequency domains



W. Heisenberg

- The product of the "widths" in the two domains is constant.
- For a time domain record with duration T , the resolution in the frequency domain is proportional to $1/T$.
- Perfect resolution in frequency requires infinite record length in time.
- Infinite bandwidth in frequency is needed to represent a time function exactly.

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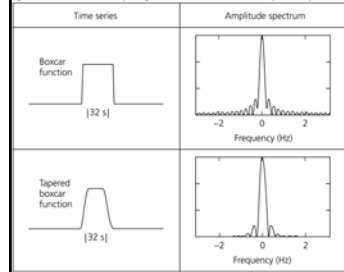
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Tapered boxcar functions

$$W(t) = \frac{1}{2} \left[1 + \cos \frac{\pi(t+T-T_1)}{T_1} \right] \text{ for } -T < t < -T+T_1 \\ = \frac{1}{2} \left[1 + \cos \frac{\pi(t-T+T_1)}{T_1} \right] \text{ for } T-T_1 < t < T$$

Figure 6.3-10: Effects of tapering a boxcar function on the amplitude spectrum.



The side lobes for the tapered window are reduced, but the central peak is less sharp.

Similarly, band-pass filters are often tapered in the frequency domain.

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Cross-correlation

$$C(L) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) f(t+L) dt$$

- $C(L)$, the cross-correlation of $x(t)$ and $f(t)$, measures the similarity between $f(t)$ and the later portions of $x(t)$ by shifting $f(t)$ by different lag times, L , and evaluating the integral of the product as a function of L .
- We often set T to an appropriate value, due to finite length of the data.

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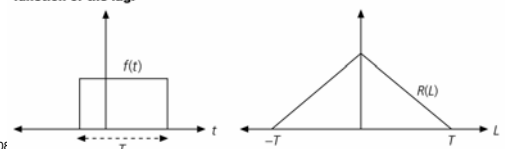
Auto-correlation

- A special case of the cross-correlation is the auto-correlation.

$$R(L) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} f(t) f(t+L) dt$$

- The auto-correlation is maximum at zero lag, and is an even function of the lag.

Figure 6.3-12: The auto-correlation is maximum at zero lag and is an even function of the lag.



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Auto-correlation and amplitude spectrum

$$R(L) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} f(t)f(t+L)dt$$

- Can be expanded using the inverse Fourier transform

$$\begin{aligned} R(L) &= \lim_{T \rightarrow \infty} \frac{1}{2\pi T} \int_{-T/2}^{T/2} f(t) \left[\int_{-\infty}^{\infty} F(\omega) e^{i\omega(t+L)} d\omega \right] dt \\ &= \lim_{T \rightarrow \infty} \frac{1}{2\pi T} \int_{-\infty}^{\infty} F(\omega) e^{i\omega L} \left[\int_{-T/2}^{T/2} f(t) e^{i\omega t} dt \right] d\omega \\ &= \lim_{T \rightarrow \infty} \frac{1}{2\pi T} \int_{-\infty}^{\infty} F(\omega) F(-\omega) e^{i\omega L} d\omega \\ &= \lim_{T \rightarrow \infty} \frac{1}{2\pi T} \int_{-\infty}^{\infty} |F(\omega)|^2 e^{i\omega L} d\omega \end{aligned}$$

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Auto-correlation and amplitude spectrum

- If we define the power spectrum, a normalized version of the amplitude spectrum

$$P(\omega) = \lim_{T \rightarrow \infty} \frac{1}{T} |F(\omega)|^2$$

- Then the auto-correlation is the inverse Fourier transform of the power spectrum:

$$R(L) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(\omega) e^{i\omega L} d\omega$$

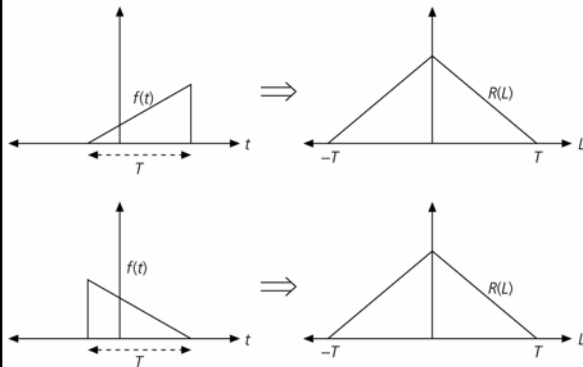
- As a result, the auto-correlation of a function contains information only about its amplitude spectrum, but not about its phase.

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Figure 6.3-13: A function has the same auto-correlation if it is reversed in time.



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Cross-correlation and convolution

$$y(L) = \int_{-\infty}^{\infty} x(t)f(L-t)dt = x(t) * f(t)$$

$$C(L) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)f(t+L)dt = x(t) \bullet g(t)$$

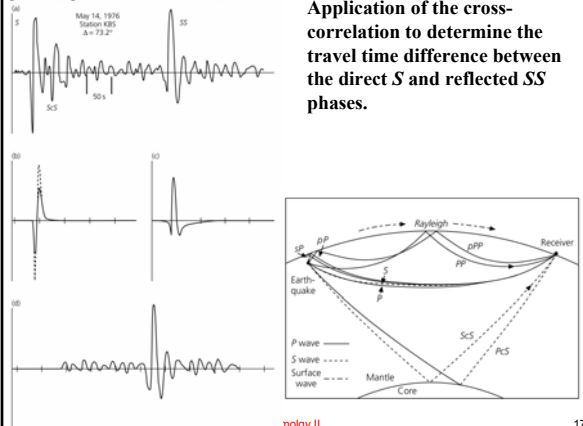
The **cross-correlation** is similar in nature to the **convolution** of two functions. Whereas **convolution** involves reversing a signal, then shifting it and multiplying by another signal, **correlation** only involves shifting it and multiplying (no reversing).

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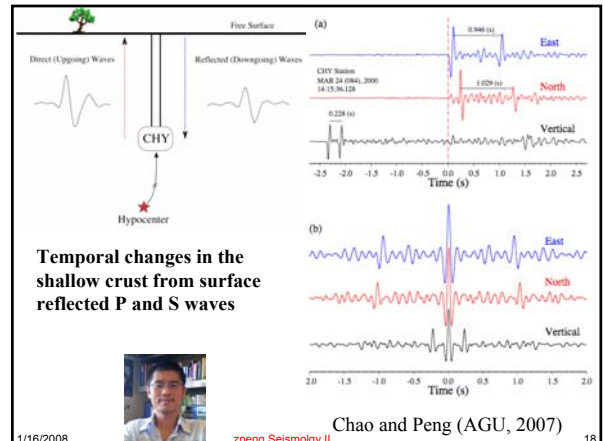
Figure 6.3-11: Using cross-correlation to find a differential SS-S travel time.



Application of the cross-correlation to determine the travel time difference between the direct S and reflected SS phases.

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Temporal changes in the shallow crust from surface reflected P and S waves



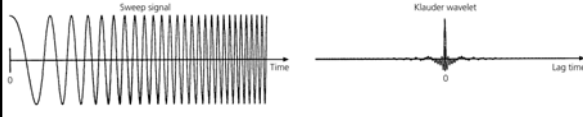
Chao and Peng (AGU, 2007)

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Figure 3.3-30: Auto-correlation of a Vibroseis sweep signal.

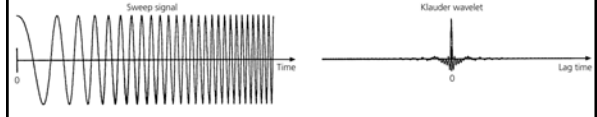


If $w(t)$ is a long signal, use cross-correlation.

The cross-correlation quantifies similarities between two time series $f(t)$ and $g(t)$:

$$c(L) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^T f(t+L)g(t)dt$$

Figure 3.3-30: Auto-correlation of a Vibroseis sweep signal.



If $w(t)$ is a long signal, use cross-correlation.

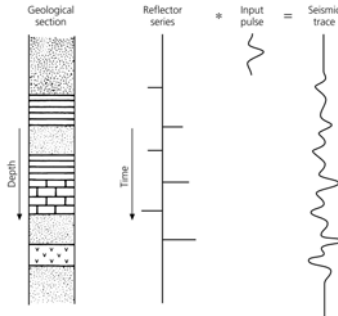
The cross-correlation quantifies similarities between two time series $f(t)$ and $g(t)$:

$$c(L) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^T f(t+L)g(t)dt$$

For example, the cross-correlation of $w(t)$ with itself (called auto-correlation) is:

$$a(L) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^T f(t+L)f(t)dt \quad (\text{which is always maximum at zero lag})$$

Figure 3.3-28: Reflection seismogram as the convolution of a source pulse with a reflector series.



Convolution of $w(t)$ and $r(t)$:

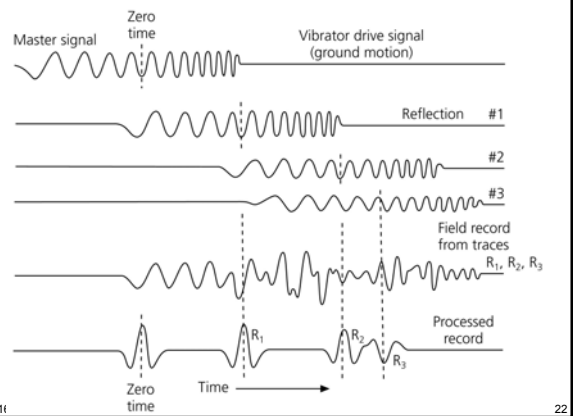
$$s(t) = w(t) * r(t) \equiv \int_{-\infty}^{\infty} w(t-\tau)r(\tau)d\tau$$

$$S(\omega) = W(\omega)R(\omega)$$

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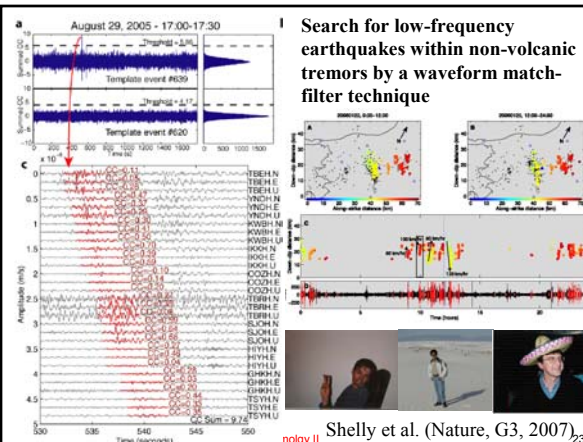
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Figure 3.3-31: Analysis of a Vibroseis record.



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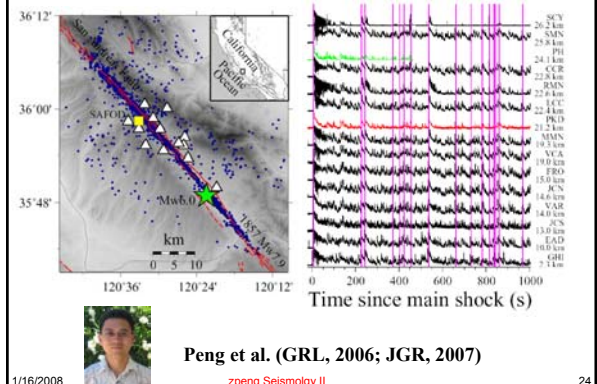
Search for low-frequency earthquakes within non-volcanic tremors by a waveform match-filter technique

Shelly et al. (Nature, G3, 2007)

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Many aftershocks are missing immediately after an mainshock



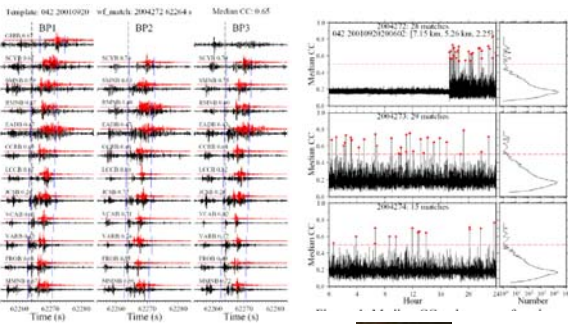
Peng et al. (GRL, 2006; JGR, 2007)

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Search for missing aftershocks using match-filter technique



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What we have learned today

- Linear systems
 - Basic models
 - Convolution and deconvolution modeling
 - Finite length signals
 - Correlation

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Next time

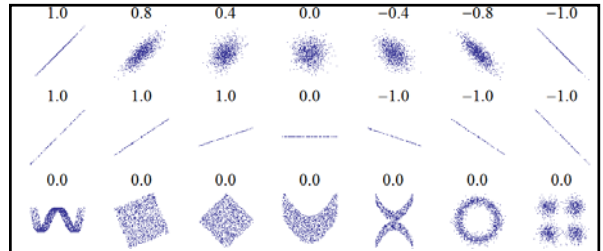
- Discrete time series and transforms

Reading: Stein and Wyession Chap. 6.4

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