EAS 8803 - Seismology II
Lec#3: Linear Systems

Dr. Zhigang Peng, Spring 2008

Figure 6.2-3: Amplitude spectra for the body and surface wave segments from a large earthquake.

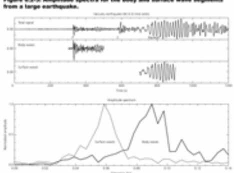
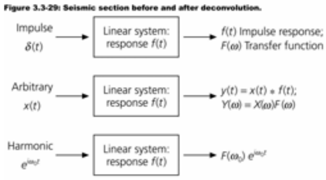


Figure 3.3-29: Seismic section before and after deconvolution.



Last Time

- Fourier transforms
  - From Fourier Series to Fourier Transforms
  - Properties of Fourier Transform
    - Linear
    - Shifting in time equals timing a phase term in frequency domain
    - The Fourier transform of the derivative of a time function is found by multiplication ($i\omega$)
  - Delta function
    - Time and frequency domain representation

Today's Outline

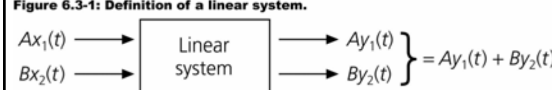
- Linear systems
  - Basic models
  - Convolution and deconvolution modeling
  - Finite length signals
  - Correlation

Reading: Stein and Wysession Chap. 6.3

Linear Systems

- A "**system**" is a general representation of any device or processes that takes an input signal and modifies it.
- A "**linear system**" is is defined by the following diagram, and is previously referred as the principle of *superposition*.

Figure 6.3-1: Definition of a linear system.

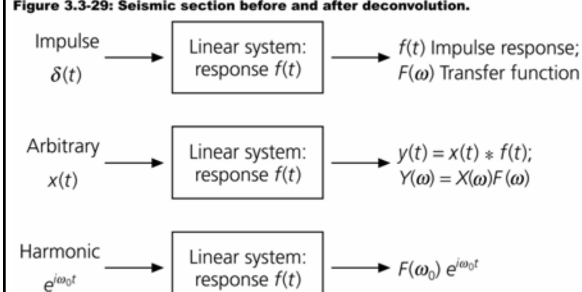


Linear Systems

- The earth generally behaves as a "*linear system*" when transmitting seismic waves.
- Hence, linear system models are used in a wide variety of seismological applications.
- Fourier analysis is a natural tool for studying linear systems because Fourier transform has the same linear properties.
- Can you think of any cases when the Earth is behaving as a "nonlinear system"?

Impulse Response of a Linear System

Figure 3.3-29: Seismic section before and after deconvolution.



Linear Systems

- The output spectrum of an arbitrary input signal

$$Y(\varpi) = X(\varpi)F(\varpi)$$

- The output in the time domain $y(t)$ can be found

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\varpi)F(\varpi)e^{i\varpi t}d\varpi$$

- For the impulse $x(t)=\delta(t)$, $X(\varpi)=1$, $y(t)=f(t)$
- For a harmonic input signal $x(t)=e^{i\varpi_0 t}$
- The transform is the delta function in frequency domain $X(\varpi)=2\pi\delta(\varpi-\varpi_0)$. The output is

$$Y(\varpi) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi\delta(\varpi-\varpi_0)F(\varpi)e^{i\varpi t}d\varpi = F(\varpi_0)e^{i\varpi_0 t}$$

Relation between the input/output and the impulse response

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\varpi)F(\varpi)e^{i\varpi t}d\varpi$$

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} x(\tau)e^{-i\varpi\tau}d\tau\right]\left[\int_{-\infty}^{\infty} x(\tau')e^{-i\varpi\tau'}d\tau'\right]e^{i\varpi t}d\varpi$$

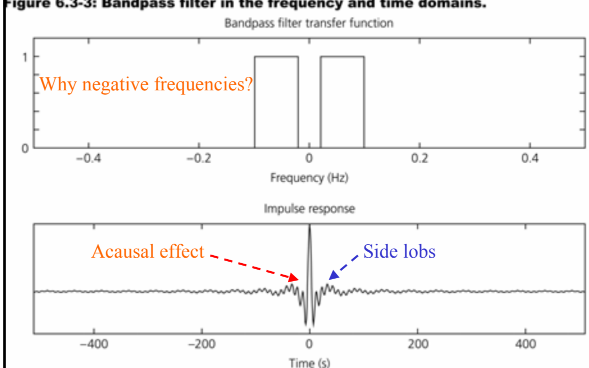
$$y(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\tau)f(\tau')\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\varpi(t-\tau'-\tau)}d\varpi\right]d\tau d\tau'$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\left[\int_{-\infty}^{\infty} f(\tau')\delta(t-\tau'-\tau)d\tau'\right]d\tau$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)f(t-\tau)d\tau \dashrightarrow y(t) = x(t)*f(t)$$

Convolution

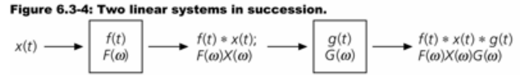
Figure 6.3-3: Bandpass filter in the frequency and time domains.



Convolution and deconvolution modeling in seismology

- Linear system ideas are pervasive in seismology.
- If a signal $x(t)$ goes through two linear systems in succession with impulse response $f(t)$ and $g(t)$, the output is either a convolution in the time domain, or the product of the transfer functions in the frequency domain.

Figure 6.3-4: Two linear systems in succession.



Convolution and deconvolution modeling in seismology

$$u(t) = x(t) * g(t) * i(t)$$

Figure 6.3-5: Seismogram as the convolution of the source, structure, and instrument signals.

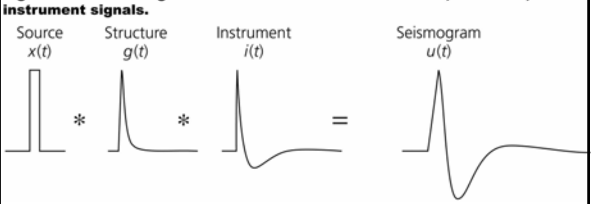
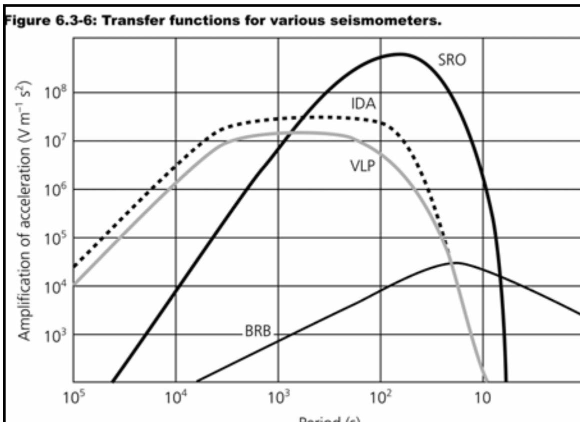
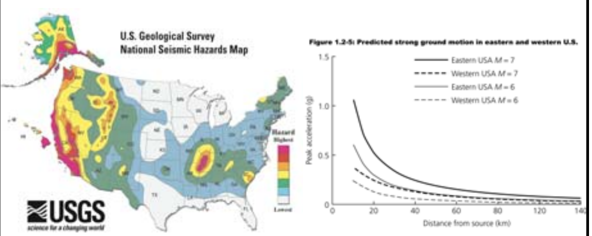


Figure 6.3-6: Transfer functions for various seismometers.



Response of a system in space by convolutions



The Green's function

- The displacement at a point $\boldsymbol{x}$ and time t is

$$u(x,t)=\iint G(x-x';t-t')f(x',t')dt'dV'$$

- Where $G(x-x';t-t')$ is the Green's function, the impulse response to a source at position $\boldsymbol{x}'$ and time t', and $f(x',t')$ is the distribution of the seismic sources.
- In a general medium

$$u(x,t)=\iint G(x,t;x',t')f(x',t')dt'dV'$$

Inverse filter

- We assume that a seismogram $s(t)$ results from convolution of a source pulse $w(t)$, and an earth structure operator $r(t)$.

$$s(t)=w(t)*r(t) \qquad S(\varpi)=W(\varpi)R(\varpi)$$

- We can create an inverse filter

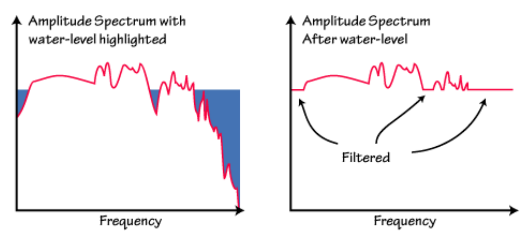
$$w^{-1}(t)*w(t)=\delta(t)$$

- The Fourier transform of the inverse filter is just $1/W(\omega)$, so the deconvolution can be done by dividing the Fourier transforms

$$S(\varpi)/W(\varpi)=R(\varpi)$$

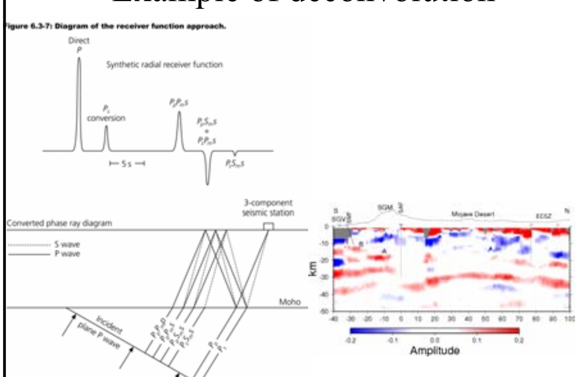
Water-level deconvolution

- For $S(\varpi)/W(\varpi)=R(\varpi)$
- What happens if $W(\omega)$ is very small?



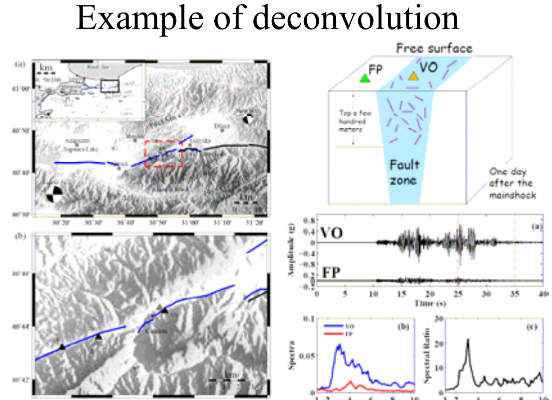
http://eqseis.geosc.psu.edu/~cammon/HTML/RftnDocs/seq01.html

Example of deconvolution



Zhu (EPSL, 2000)

Example of deconvolution



From Chunquan Wu

What we have learned today

- Linear systems
  - Basic models
  - Convolution and deconvolution modeling

Next time

- Finite length signals
- Correlation
- Discreet time series and transforms

Reading: Stein and Wysession Chap. 6.4