PROBLEM SET 1: RANDOM VARIABLES

EAS 6134: Inverse Methods and Data Analysis in EAS

Assigned: 1/24/22
Due: 1/31/22

NAME: _____

OTHERS CONSULTED: _____

Adapted from UCSD 223A
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- Please be neat and organized! Once you have found a way to the answer, please rewrite it in an orderly fashion so that others can follow your steps, and put a box around your final solution when appropriate.

- Include this page as the cover, listing all who helped with this set including me in the "Others consulted" line.

- Show all of your work.

- An answer with incorrect or absent units will be considered wrong.

- For electronic submission to Canvas, please create a single PDF of your written work. This can be done with most smartphones using tools such as *Google Drive*, *Adobe Scan*, or *Scanner App* (GoogleDrive tested, but any method acceptable).

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1. Creating usable and informative graphics:

   (a) Find an example of what you feel is a highly informative graphic from your literature. Explain why you feel this is the case.

   (b) Now, do the same, but find one that you feel misses the point, or is otherwise difficult to interpret, and explain why.

2. Consider the model for the magnetic field reversals described at the beginning of class. We can consider time to be broken up into blocks Δ long, and within each block we assume that there is a certain probability of reversal p that is independent of anything that occurs before (completely chaotic).

   (a) What is the probability of getting a period $N\Delta$ with only reversal at the end?

   (b) What is the probability of getting no reversal between the times $N_1\Delta$ and $N_2\Delta$?

   (c) What is the probability if $N_1 = 1$ and $N_2 \Rightarrow \infty$?

   (d) Evaluate the probability of an un-reversed interval longer than T for the case p/Δ constant, with Δ going to 0.

3. Suppose we have a sine wave:

$$x(t) = A\cos(t) \tag{1}$$

   and creates a random variable by sampling x at random times t. What is the PDF of this random variable? (This should be solvable by first describing the distribution of x across t.)

4. The Cauchy cumulative distribution function is

$$F(x) = \frac{1}{2} + \frac{1}{\pi}\tan^{-1}(x), \qquad -\infty < x < \infty \tag{2}$$

   (a) Show that this is a CDF.

   (b) Find the associated PDF.

   (c) Find x such that $P(X > x) = 0.1$.

5. Show that if A and B are independent, then

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A)\Pr(B) \tag{3}$$

6. Sometimes a random variable comes from a mixture of two or more distributions. Consider the PDF of the random variable X distributed as

$$X \approx \frac{3}{4}N(0, 1) + \frac{1}{4}N(1, 0.3) \tag{4}$$

   where $N(m, \sigma)$ is the Normal (Gaussian) distribution, with mean m and standard deviation σ, for which the PDF is

$$\phi(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(x-m)^2/2\sigma^2}. \tag{5}$$

   (a) Plot the PDF.

   (b) find the first four moments of the PDF (1: the mean, 2: the variance, 3: the skewness, and 4: the flatness).

   (c) finally, find the median, the interquartile distance, the mode, and the standard deviation about the mean.