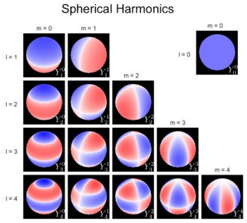
EAS 4801 - Planetary Sound
Lec #5: Standing waves

Dr. Zhigang Peng
01/15/2020
Spring 2020

U.S. Navy F/A-18 approaching the sound barrier. The white cloud forms as a result of the supersonic expansion fans dropping the air temperature below the dew point.[1][2]

Spherical Harmonics



Update on the Taal Volcanic Eruption

IN PHOTOS: A fissure is seen along the national road in Brgy. Sinisian East, Lemery, Batangas on Tuesday Jan. 14 due to constant earthquakes and the eruption of Taal Volcano. I 📷NIÑO JESUS ORBETA/PDI

Read: inq.news/Taalfissure

♡ 1,404 3:15 AM - Jan 14, 2020

https://www.npr.org/2020/01/15/796541789/in-philippines-volcano-is-quieter-but-officials-renew-warnings-for-people-to-lea

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2020 Taal Volcano eruption

Taal Volcano's January 12, 2020 explosion

Volcano	Taal Volcano
Date	January 12, 2020 – ongoing
Type	Phreatic eruption, strombolian eruption
Location	Batangas, Calabarzon, Philippines
Impact	At least 1 injured by ashfall[1]

https://en.wikipedia.org/wiki/2020_Taal_Volcano_eruption
https://www.gns.cri.nz/Home/Learning/Science-Topics/Volcanoes/Types-of-Volcanoes-Eruptions

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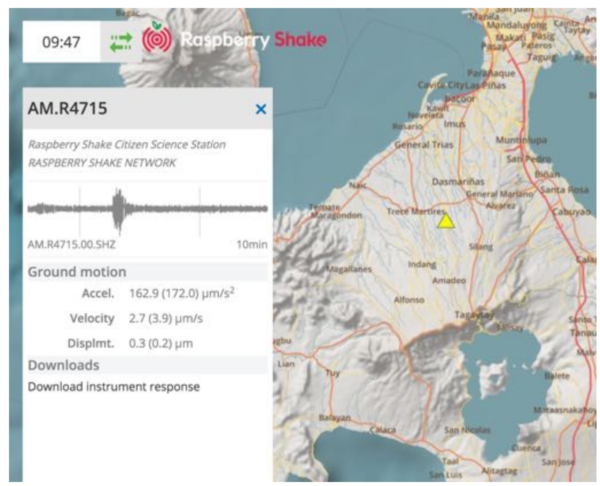
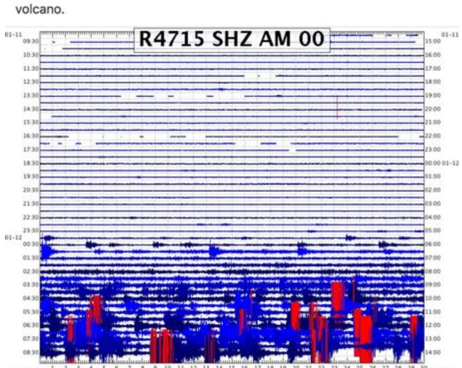


https://raspberryshake.net/stationview/#?net=AM&sta=R4715

Steven Jaume
January 12 at 9:28 AM

Helicorder image of Raspberry Shake near Taal Volcano in the Philippines. Note increase in activity over the past few hours - Taal has started erupting. Taal Volcano is south of the Manila metropolitan area and it looks as if they are in the process of evacuating the area near the volcano.



How to request data from the raspberry shake sensor

SeisComP3 FDSNWS DataSelect - URL Builder

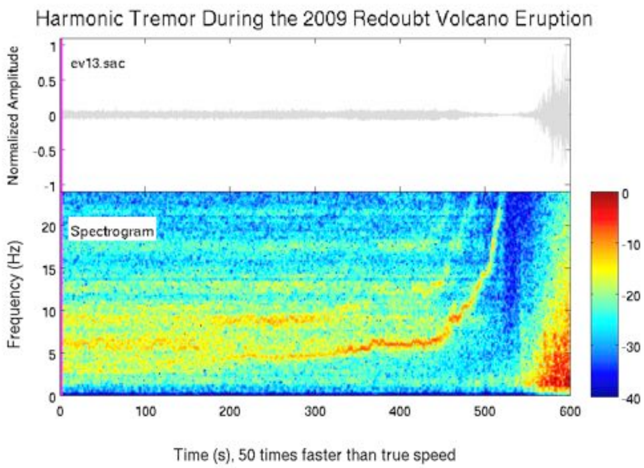
Time constraints
Start Time 2020-01-12
End Time 2020-01-13

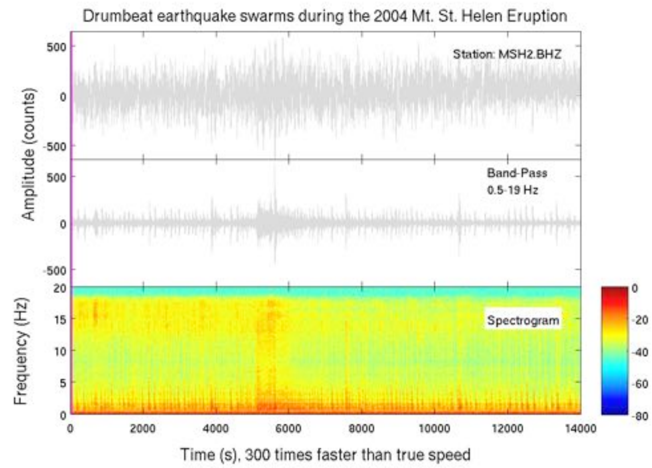
Channel constraints
Network AM
Station R4715
Location 00
Channel SHZ

Service specific constraints
Quality B
Minimum Length (s) 0.0
Longest Only
Authentication

Output control
Format miniseed
No Data 404

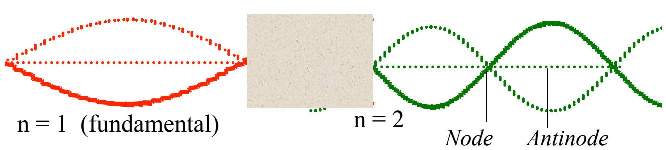
https://data.gempa.de/fdsnws/dataselect/1/builder
https://raspberryshake.net/stationview/



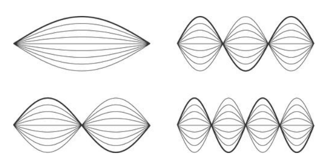


Standing Waves: 2 Fixed Ends

When a guitar string of length L is plucked, only certain frequencies can be produced, because only certain wavelengths can sustain themselves. Only standing waves persist. Many harmonics can exist at the same time, but the fundamental (n = 1) usually dominates. As we saw in the wave presentation, a standing wave occurs when a wave reflects off a boundary and interferes with itself in such a way as to produce nodes and antinodes. Destructive interference always occurs at a node. Both types occur at an antinode; they alternate.



Normal Modes of a String



General solutions

$u(x,t) = A\cos(\omega t - kx) + B\cos(\omega t + kx)$

Boundary conditions

$A\cos\omega t + B\cos\omega t = 0$

$A\cos(\omega t - kL) + B\cos(\omega t + kL) = 0$

Standing wave solutions

$u_n(x,t) = U_n(x,\omega_n)\cos(\omega_n t)$

$= \sin(n\pi x / L)\cos(\omega_n t)$

Wave equation

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u(x,t)}{\partial t^2}.$$

The boundaries make the modes! No boundaries, no discrete modes!

(an infinitely long string would have continuous modes at all frequencies)

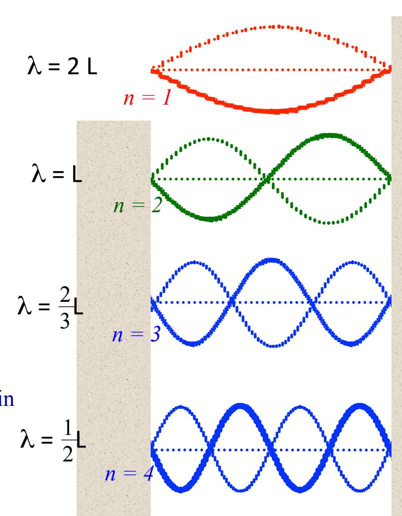
Wavelength Formula: 2 Fixed Ends
(string of length L)

Notice the pattern is of the form:

$$\lambda = \frac{2\,L}{n}$$

where n = 1, 2, 3, ...

Thus, only certain wave-lengths can exists. To obtain tones corresponding to other wavelengths, one must press on the string to change its length.



Vibrating String Example

Schmedrick decides to build his own ukulele. One of the four strings has a mass of 20 g and a length of 38 cm. By turning the little knobby, Schmed cranks up the tension in this string to 300 N. What frequencies will this string produced when plucked? *Hints:*

1. Calculate the string's mass per unit length, μ: 0.0526 kg / m
2. How the speed of a wave traveling on this string using the formula $v = \sqrt{F / \mu}$: 75.498 m / s
3. Calculate several wavelengths of standing waves on this string:
   0.76 m, 0.38 m, 0.2533 m
4. Calculate the corresponding frequencies:
   99 Hz, 199 Hz, 298 Hz

Real Example

INVITED ARTICLE
Tibetan singing bowls
Denis Terwagne[1] and John W M Bush[2]
Published 1 July 2011 • 2011 IOP Publishing Ltd & London Mathematical Society
Nonlinearity, Volume 24, Number 8

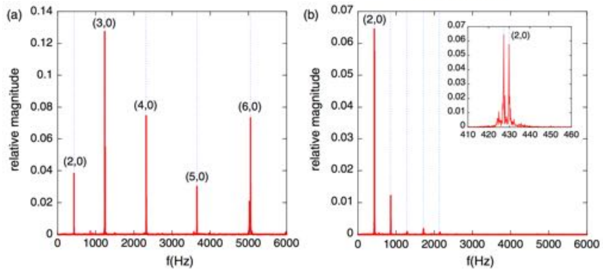


Figure 3. *(a)* Frequencies excited in the bowl Tibet 4 when struck with a wooden mallet. The different peaks correspond to the natural resonant frequencies of the bowl, and the associated deformation modes (n, m) are indicated. *(b)* Excited frequencies of the bowl Tibet 4 when rubbed with a leather mallet. The first peak corresponds to the deformation mode (2,0) and subsequent peaks to the harmonics induced by mode lock-in. In the inset, a magnification of the first peak provides evidence of its splitting due to the asymmetry of the bowl.

https://iopscience.iop.org/article/10.1088/0951-7715/24/8/R01/pdf

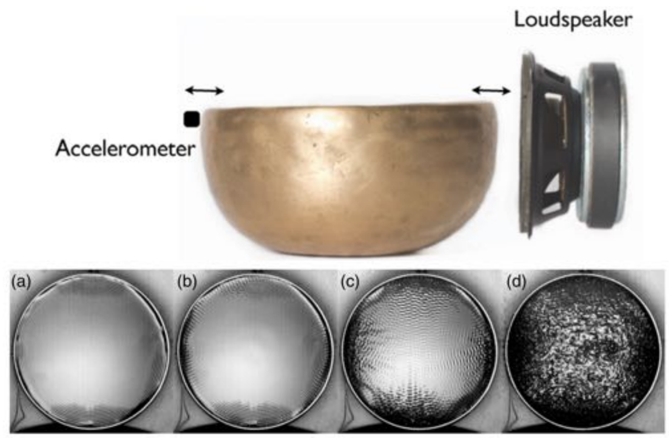
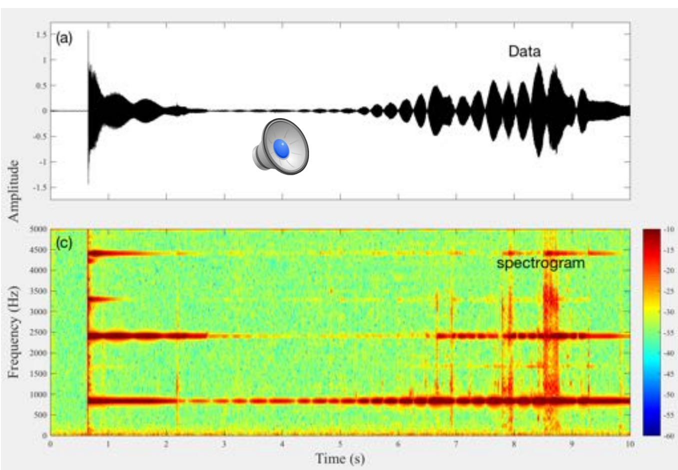
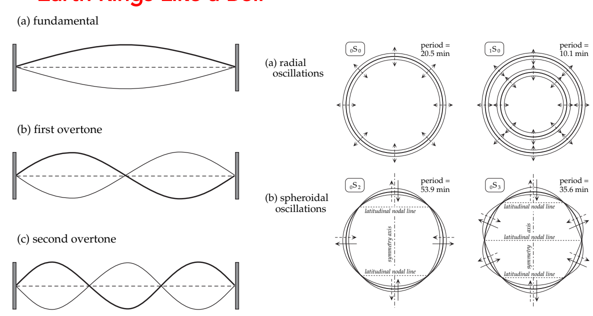


Figure 7. Evolution of the surface waves in Tibet 1 bowl filled with water and excited with a frequency $f = 188$ Hz corresponding to its fundamental mode (2,0). The amplitude of deformation is increasing from left to right: *(a)* $\Gamma = 1.8$, *(b)* $\Gamma = 2.8$, *(c)* $\Gamma = 6.2$, *(d)* $\Gamma = 16.2$.



Free Oscillations of Earth

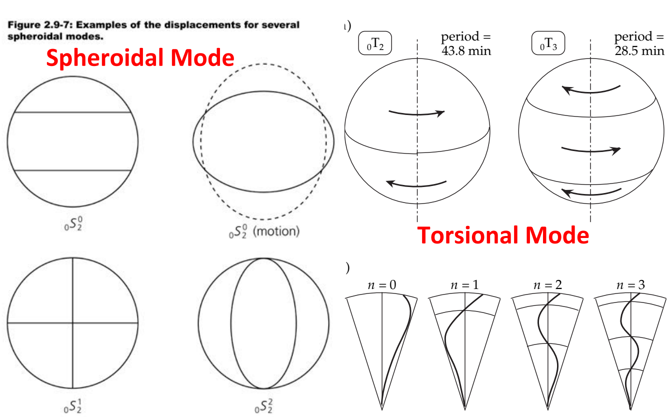
Earth Rings Like a Bell



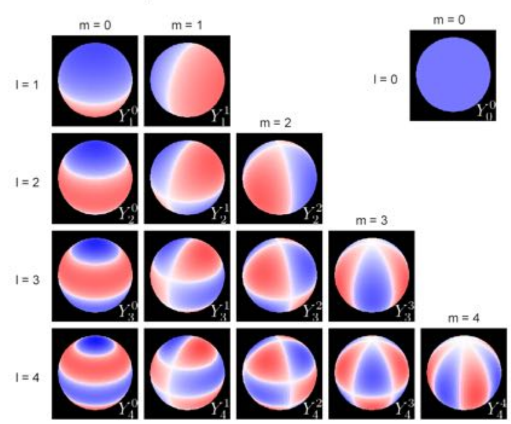
Standing Waves

Pulsing In-and-Out

Free Oscillations of Earth



Spherical Harmonics



Miaki Ishii

https://saviot.cnrs.fr/terre/index.en.html