Appendix C: Specifying and Using Channel Response Information

Contributed by C.R. Hutt

Introduction

SEED volumes usually use complex-valued functions of frequency in response functions. Usually, these functions will not be single expressions, but rather the products of several expressions. Most seismic systems can be regarded as cascades of stages — for example, a seismometer, followed by an amplifier, followed by an analog filter, followed by an analog/ digital converter, followed by a digital filter. A blockette's stage sequence number shows the order of the stages, as shown in figure 1 below:

Figure 1: Example of a sequence of stages.

Before the age of high speed digital computers and digital signal processing (DSP) chips, all low-pass filtering (for the purpose of preventing aliasing) was performed in the analog stages before digitizing. The digitizer would operate at a fairly low sample rate equal to the sample rate being recorded. Typically, the corner frequency of the low-pass filter would be 1/8 to 1/20 of the sample rate (1/4 to 1/10 of the Nyquist frequency). Therefore, the low-pass anti-alias filter response would typically begin to attenuate at frequencies well within the band of interest.

Modern seismic data acquisition systems make use of over-sampling and decimation (to grossly over-simplify the inner workings of high resolution ADC's) to achieve high resolution. This technique relaxes the analog anti-alias filtering requirement and moves the low-pass filtering job into the digital domain. Decimation (sample rate reduction) must be preceded by sufficient low-pass filtering to prevent aliasing at the new lower sample rate. Many modern high resolution ADC's include two or more stages of Finite Impulse Response (FIR) filters to accomplish this task. These may be followed by further low-pass filter and decimate stages within the data acquisition computer to derive lower sample rate data streams (such as deriving Long Period data from Broadband data).

FIR filters are simply weighted averages of some number of data samples — the "weights" are the coefficients specified in Blockette (54) (for a "D" type stage). FIR filters are usually designed to approximate a "boxcar" response. That is, they typically have a very flat in-band response and a sharp, steep cut-off at their corner frequency, which may be set at 70% to 90% of the Nyquist frequency. In-band ripple is usually only a few percent. Also, FIR filters are usually designed to have linear phase, and the data acquisition systems usually time-tag the data so that the phase shift appears to be nearly zero.

All of this means that the average data user probably doesn't need to correct for the effect of such FIR filters. Examples of some common FIR filter amplitude responses for 20 sps data are shown in Figures 2 through 5 following. Note that in-band ripple can be several percent, but corner frequencies (-3 db points) are usually very close to the Nyquist frequency (which is 1/2 the sample rate). Also note that stop band gain can vary significantly: from -75 db to -120 db in these three examples.

It was previously stated that modern data acquisition systems using digital FIR filters usually time tag the data so that the filter delay (phase lag) appears to be nearly zero. Figure 2.B. contains a plot of the phase shift that results when the data are time tagged in such a way as to correct precisely for the theoretical filter delay (which is 1.625 seconds in this case). As is stated later in this Appendix, Blockette [57] should always be used when specifying a digital filter to completely describe how the time tag is applied. Some data acquisition systems (those installed through August 1992 by the USGS) correct for the FIR filter delays, resulting in near-zero phase shift, but did not specify this in a Blockette [57]. A data user may take the absence of Blockette [57] to mean that there exists a phase lag in the data that is really not there in these systems. That is, a Fourier transform of the FIR coefficients would indicate a pure delay, when in fact there is really no delay. The absence of Blockette [57] in USGS-supplied data will be remedied as soon as possible after August 1992. Anyone specifying digital filters in SEED format should always include the complete specification, including Blockette [57].

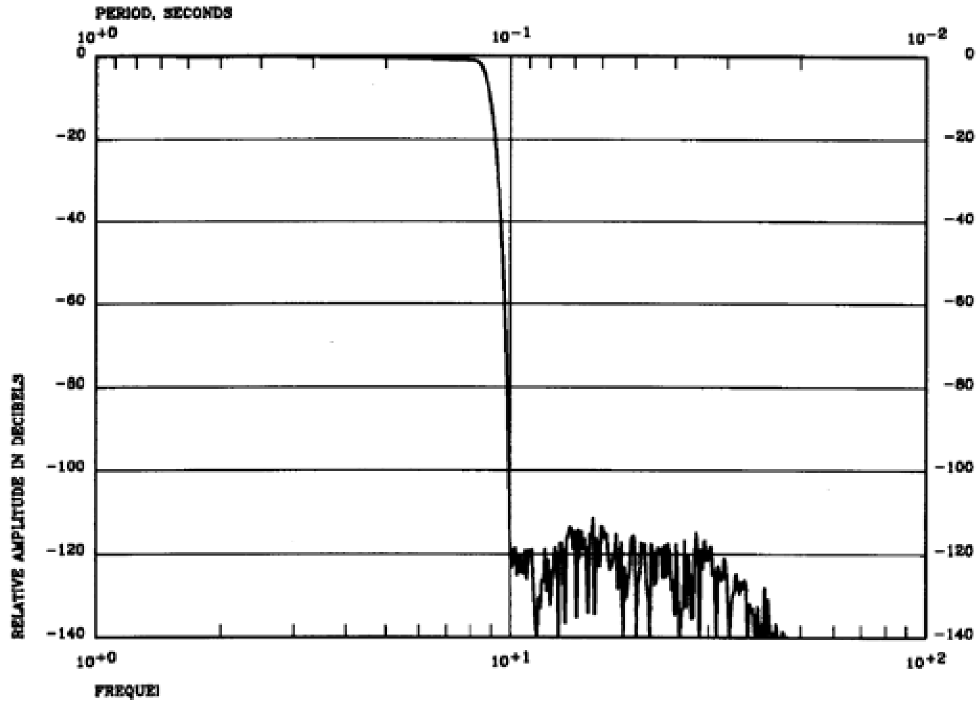

Figure 2A

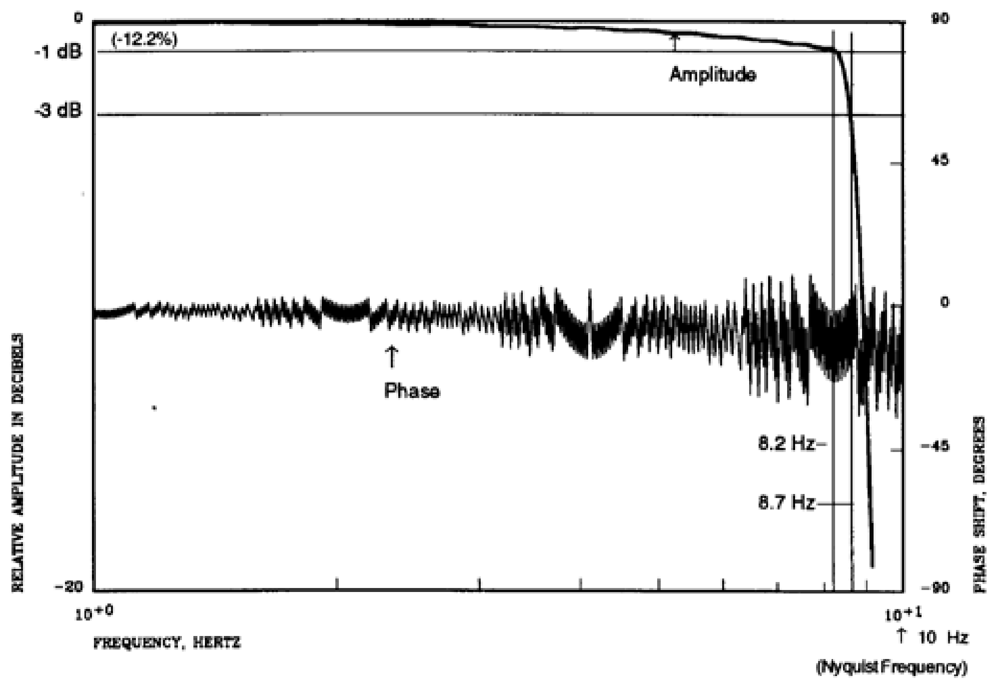

Figure 2b

Figure 2: Amplitude response of combined FIR filters used in Martin-Mariette digitizers of IRIS/USGS IRIS-2 systems, 20 sps (BB) data. Gain has been normalized to OdB at O Hz (DC). Note the different scales in the two figures above.

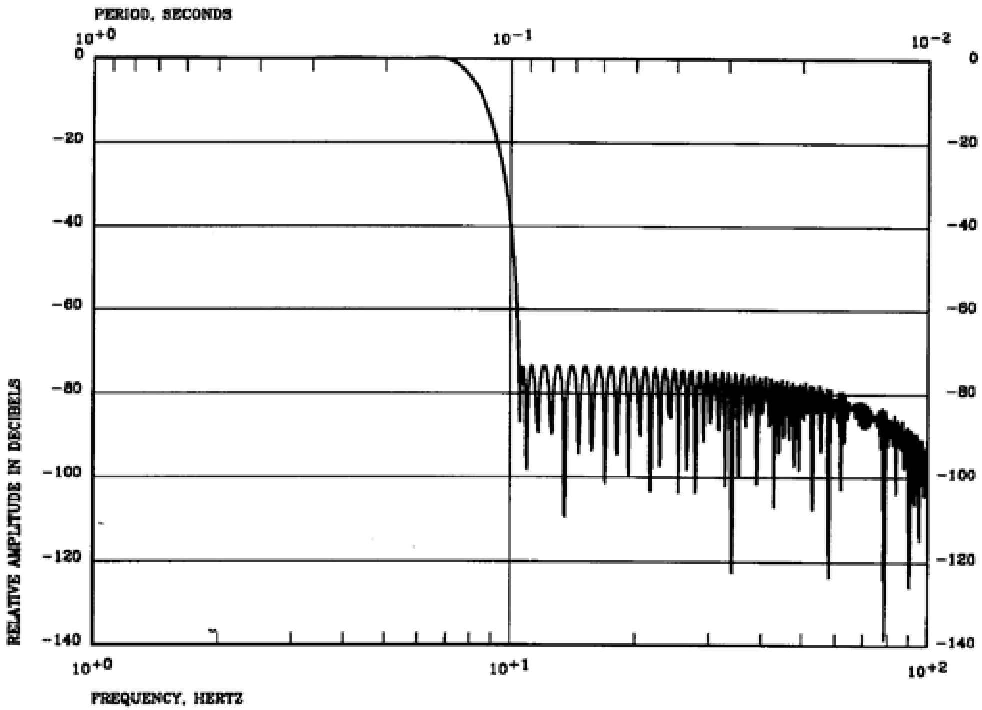

Figure 3A

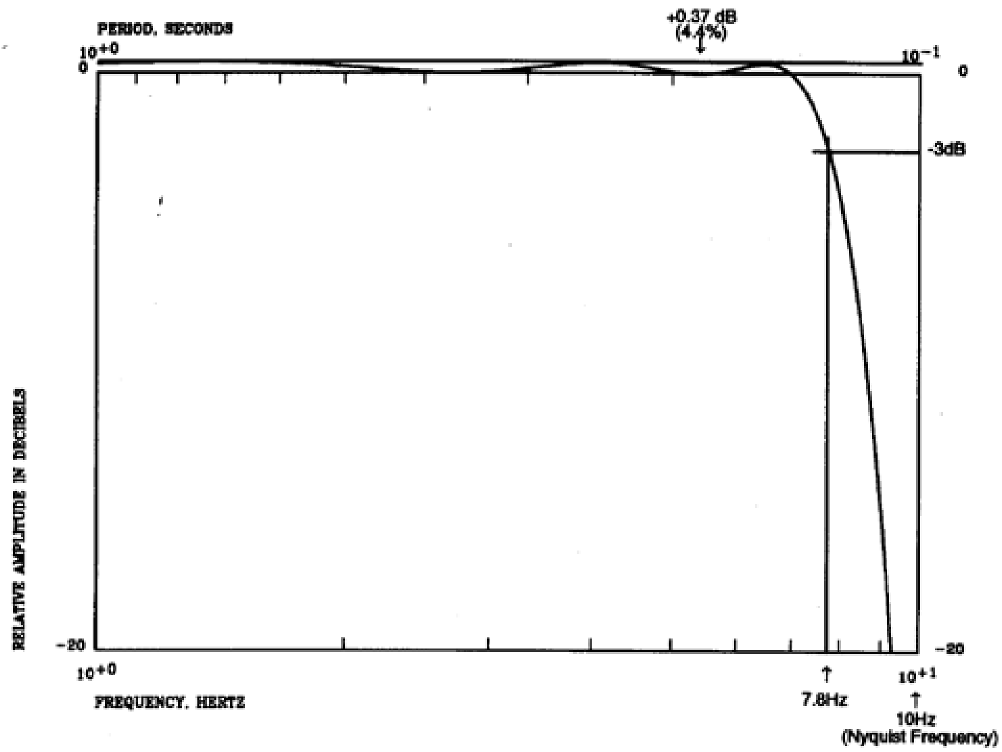

Figure 3B

Figure 3: Amplitude response of combined FIR filters used in “Quantagrator” model digitizers built by Quanterra, Inc. for 20 sps (BB) data. Gain has been normalized to Odb at OHz (DC).

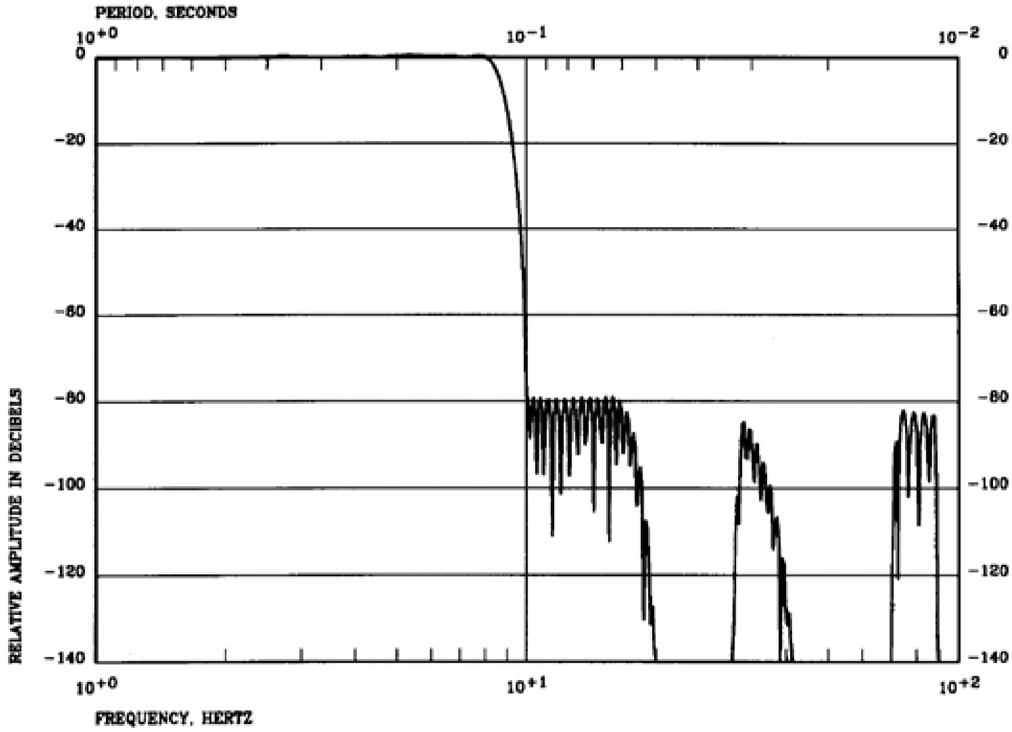

Figure 4A

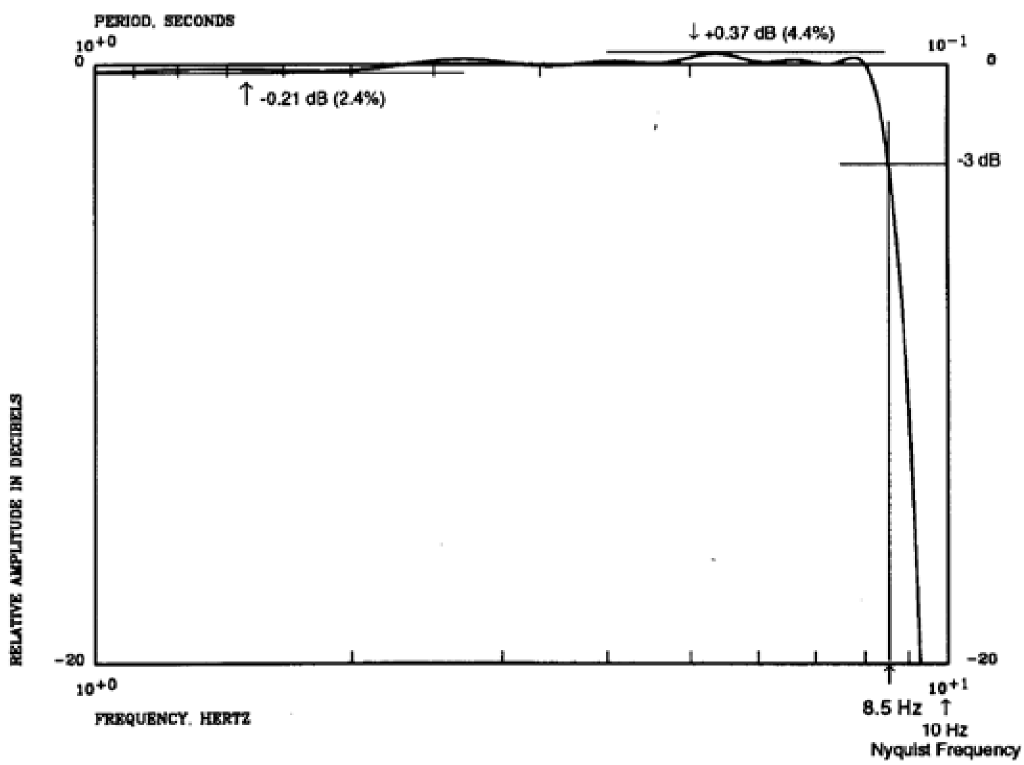

Figure 4B

Figure 4: Amplitude response of combined FIR filters used in Q380 and Q680 model digitizers built by Quanterra, Inc. for 20 sps (BB) data. Gain has been normalized to Odb at O Hz (DC).

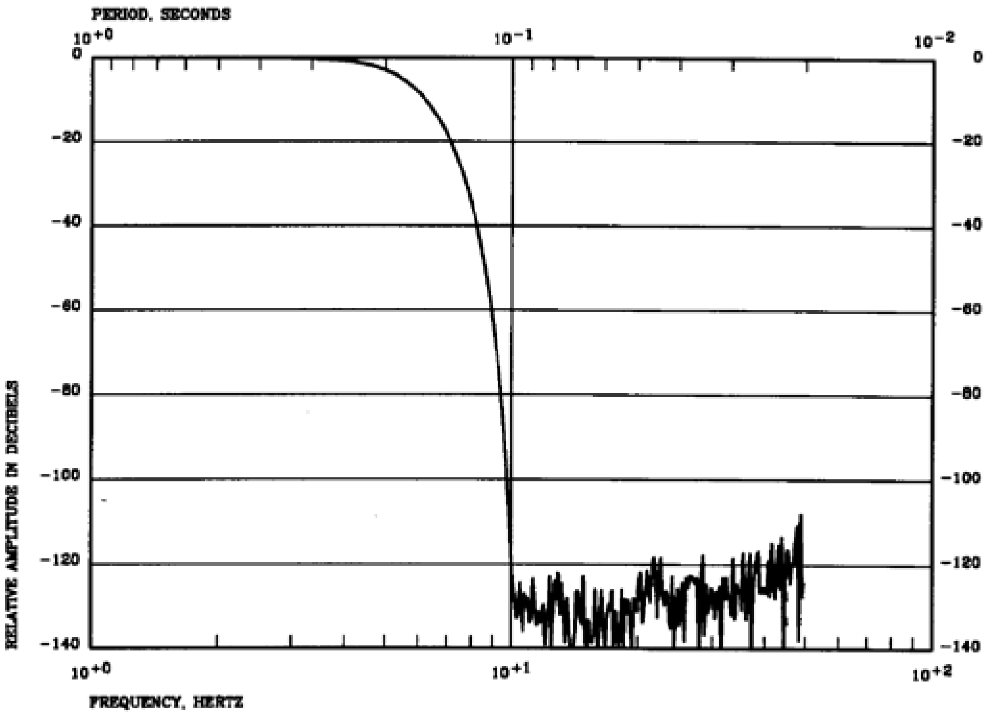

Figure 5A

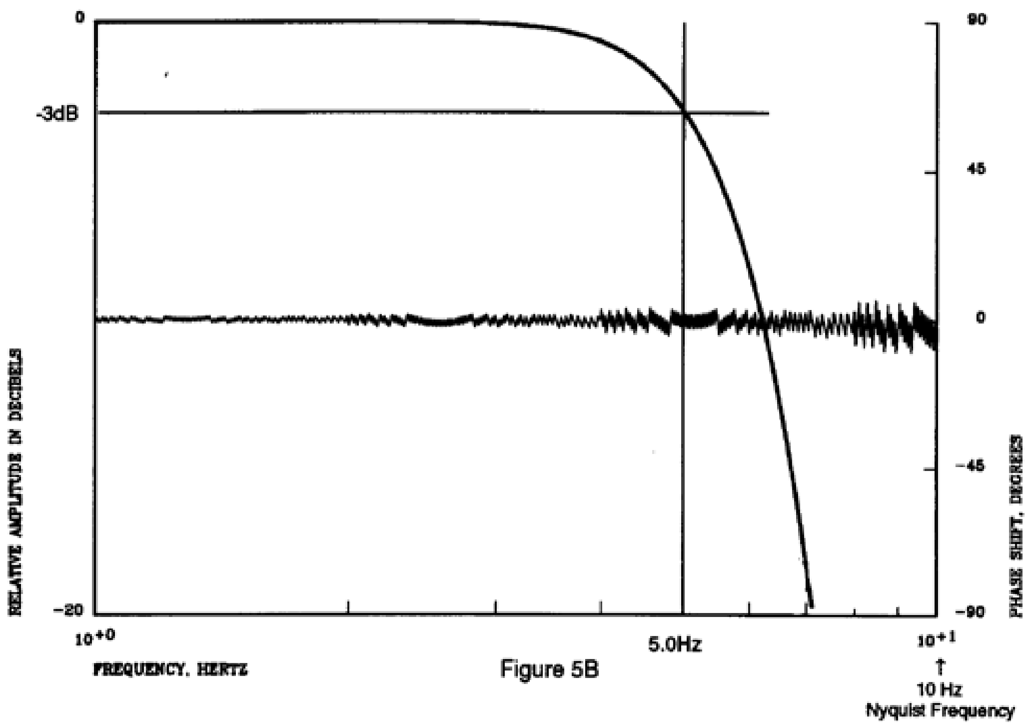

Figure 5B

Figure 5: Amplitude response of Ormsby FIR filter used in the Reftek 24-bit digitizers of the IRIS/IDA IRIS-3 systems. 20 sps (BB) data. Gain is 0 dB at 0 Hz (DC). Note the different scales in the two figures above.

In Figure 1, the seismometer response would have stage sequence number 1 and the digital filter would have stage sequence number 5. If there are K stages and the complex frequency response of the i-th one is G_i (f), the system response is:

$$\prod_{i=1}^{K} Gi\,(f) \qquad (1)$$

This appendix will show how to represent the stages (G_i's) using SEED blockettes. In Figure 1, each stage can be described by one or a combination of blockettes. Analog stages may be partially described by either Blockette [55] (Response List) or by Blockette [56] (Generic Response), but must also be described fully by using either the Poles and Zeros Blockette [53] or the Coefficient Blockette [54] along with [58] Channel Sensitivity/Gain Blockette:

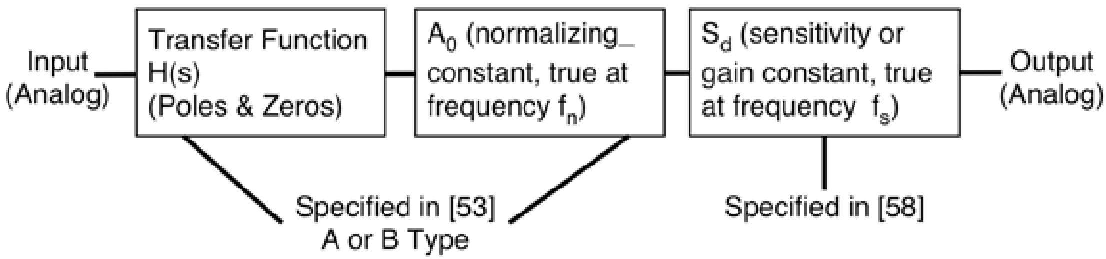

Figure 6: Example Analog Stage Using Poles and Zeros Representation

Note that A_o is chosen so that, at the normalization frequency, f_n, $|H(i2\pi f_n)|\ A_0 = 1.0$. Also note that it is most convenient, **and strongly recommended, that $f_n = f_s$.**

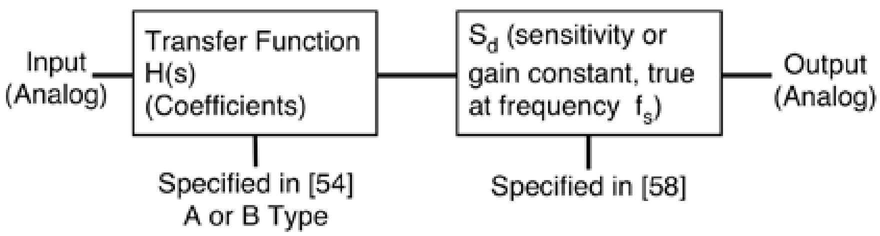

Figure 7: Example Analog Stage Using Coefficients Representation

Note that the coefficients of H(s) are chosen so that at the frequency of sensitivity f_s.$|H(i2\pi f_s)|$.=1.0. Here f_s should be equal to f_s and f_n for all previous stages in the sequence, if possible.

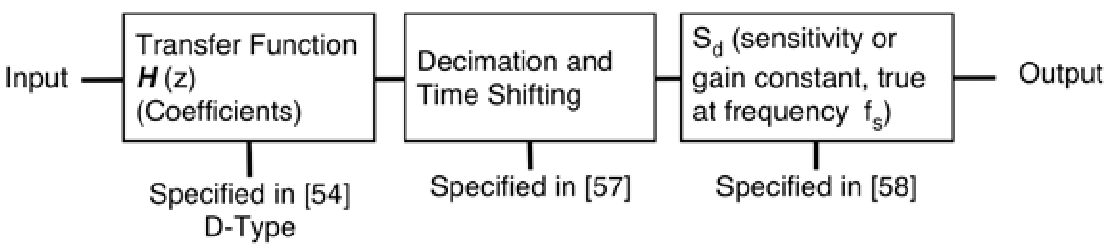

Figure 8: Example Digital Stage Using Coefficients Representation

The coefficients are chosen so that at the frequency of sensitivity f_s, $|\boldsymbol{H}(e^{2\pi i f_s \Delta t})| = 1.0$. Here, f_s should be equal to f_s and f_n for previous analog stages in the sequence, if possible. If the digital stage is a FIR filter, it is also convenient to use f_s = 0 Hz (DC), because the DC gain of a FIR filter is just the sum of the coefficients. However this should only be done if the DC gain is within 1% or 2% of the gain at f_s in previous stages.

Conventions

At any frequency, the modulus (absolute value) of the complex response function is the amplitude response of that stage. The phase of the complex response function is the phase response of that stage, with negative phase (output phase lagging the input) indicating a delay. Analog stages are represented by the Laplace transform of the linear system impulse response:

$$H(s) = \int_0^\infty h(t)\, e^{-st}\, dt \qquad (2)$$

h(t) is called the stage impulse response function, and its transform, H (s), is called the stage transfer function. H(s) may be specified in polynomial form (Blockette [54]) or in factored form (Blockette [53]).

Digital stages are represented by the Z-transform of the sampled time series corresponding to the stage impulse response:

$$\boldsymbol{H}(z) = \sum_{m=-\infty}^{\infty} h_m\, z^{-m} \qquad (3)$$

h_m is called the stage impulse response function, and its transform, $\boldsymbol{H}$ (Z), is called the stage transfer function. $\boldsymbol{H}(z)$ may be specified in polynomial form (Blockette [54], usually used for FIR filters) or in factored form (Blockette [53], usually used for IIR filters).

Normalization

For most stages, the frequency response is given in the form:

$$G(f) = S_d\; R(f) \qquad (4)$$

where R (f) is a function of frequency (usually complex-valued), specified by some combination of Blockettes [53], [54], [55], [56], and [57] (see below for which combinations are preferred for particular systems). R (f) is normalized so that $| R(f_s) | = 1.0$, where f_s is the frequency specified in Blockette [58]. S_d is the stage gain at that frequency. Using frequency response normalization helps by providing a check (you can compute G (f_s) and make sure that it is indeed S_d), and by keeping track of the response functions of analog systems.

In cases where G(f) corresponds to an analog-type stage, a Poles and Zeros type response Blockette [53] is normally used to specify this stage. In this case, R(f) is expressed in this form:

$$R(f) = A_0\, H_p(s) \qquad (5)$$

where $s = i\, 2\, \pi\, f$ or $s = i\, f$ ($i = \sqrt{-1}$) as specified below equation (6) and H_p (s) represents the transfer function ratio of polynomials specified by their roots, as in equation (6). For proper normalization, we chose A_0 such that $| R(f_s) | = 1.0$; that is $A_0 = 1/|H_p(s_s)|$, where $s_s = i 2\pi f_s \frac{\text{rad}}{\text{sec}}$ or $s_s = i\, f_s$ (depending on whether we have represented the poles and zeros of Hp in terms of radians per second or Hz).

In cases where G(f) corresponds to an analog-type stage and the coefficient representation is used, as in equation (7), then the coefficients a_j and b_j of $H_c(s)$ are chosen such that $H_c(s_s)| = 1.0$, where $s_s = i\, 2\, \pi\, f_s$ or $s_s = i\, f_s$ Hz.

When G(f) corresponds to a digital-type stage and is represented with poles and zeros, as is usually the case with IIR filters (those with feedback), we again chose $A_0 = 1/|\boldsymbol{H}_p(z_s)|$ where $\boldsymbol{H}_p(z)$ is defined as the ratio of polynomials in equation (11), and $z_s = e^{2\pi i f_s \Delta t}$, where Δt is the sample interval and f_s is specified in the stage description.

Finally, when G(f) specifies a digital-type filter and is represented with coefficients, as is usually the case with FIR filters (those without feedback), the coefficients b_n of $\boldsymbol{H}_c(z)$ in equation (9) are chosen such that $|\boldsymbol{H}_c(z_s)| = 1.0$, where z_s is defined as in the previous paragraph.

This normalization works for stages 1 through K. If Blockette [58] has a stage number of 0, SEED assumes that the sensitivity S_d given in field 4 of Blockette [58] applies to the system as a whole, at the frequency f_s given in field 5 of Blockette [58]. Note that f_s should, if possible, be equal to the normalization frequency f_n given in any of stages 1 through K. In fact, within any stage, f_s should be equal to f_n. If no other stages are specified, SEED programs should conclude that this is our total knowledge of the system response. If we specify other stages, the stage-zero sensitivity will serve as a check on the sensitivity we can arrive at by multiplying together the responses $G_1, \ldots, G_K$. In this case, the stage-zero sensitivity is not multiplied together with the gains of the other stages. Rather, the stage-zero sensitivity should be equal to the product of the gains of the other stages at frequency $f_s = f_n$. If we have not used the same frequencies, f_s and f_n, for all stages 1 through K, then we can only say that the product of the sensitivities for each stage may be approximately equal to the stage 0 sensitivity. Note that this idea is much more intuitive and easier to work with if f_s and f_n are the same for all stages.

Appendix C

A possible exception is when the stage is a low pass digital FIR filter. The stage sensitivity for a FIR stage may be stated at $f_s = 0$ Hz (DC) if the in-band ripple is less than say, 1 or 2%. The DC gain of an FIR filter is the sum of the coefficients and so is easy to calculate.

Analog Stages

The first part of any seismic sensor will be some sort of linear system that operates in continuous time, rather than discrete time. Usually, any such system has a frequency response that is the ratio of two complex polynomials, each with real coefficients. These polynomials can be represented either by their coefficients or by their roots (poles and zeros). The latter is the preferred mode, but either is acceptable.

Pole-Zero Representation for Analog Stages

The polynomials are specified by their roots. The roots of the numerator polynomial are the instrument zeros, and the roots of the denominator polynomial are the instrument poles. Because the polynomials have real coefficients, complex poles and zeros will occur in complex conjugate pairs. By convention, the real parts of the poles and zeros are negative, which leads to the form of function given below.

The fullest possible specification will utilize Blockettes [53] and [58]. Blockette [53] will specify N zeros, $r_1, r_2, \ldots, r_N$, M poles $p_1, p_2, \ldots, p_M$, a normalization factor A_0, and a reference frequency. The reference frequency is 1 radian/second if field 3 of Blockette [53] is the character A, and 1 Hz if field 3 of Blockette [53] is the character B. Blockette [58] will specify a scaling factor S_d. Then at any frequency f (in Hz), the response is:

$$G(f) = S_d\, A_0 \frac{\prod_{n=1}^{N} (s - r_n)}{\prod_{m=1}^{M} (s - p_m)} = \text{Sd A0 Hp}\,(s) \qquad (6)$$

where $s = i\,2\,\pi\,f$ if the reference frequency is 1 radian/second, and $s = i\,f$ if the reference frequency is 1 Hz.

Using two multiplicative coefficients, A_0 and S_d, in the equation above appears to be redundant, but we suggest that you partition the response by choosing A_0 so that the modulus of A_0 times the modulus of the ratio of polynomials equals 1.0 at the normalizing frequency f_n (also specified in Blockette [53]); the S_d specified in Blockette [58] is then the stage gain at that frequency, so $|\,G(f_n)\,| = S_d$. This division allows Blockette [53] to remain the same for many systems, with the small differences between them expressed by the single number S_d in Blockette [58]. This simplifies keeping track of system responses. The "frequency of sensitivity factor" in Blockette [58] (f_s) should be the same as the normalizing frequency f_n in Blockette [53].

If Blockette [53] is omitted, SEED assumes that A_0 will be 1. This would be appropriate for an amplifier with no significant departure from a fixed gain S_d, or for a stage about which nothing was known but its gain at one frequency. SEED allows these combinations of blockettes for a stage of this type: [53], [58] or [58] by itself. Blockette [58] by itself would correspond to an amplifier with a flat response.

Coefficient Representation for Analog Stages

The polynomials are specified by their coefficients. The fullest possible specification will utilize Blockettes [54] and [58]. Blockette [54] will specify N+1 numerator coefficients, $a_0, a_1, \ldots, a_N$, M +1 denominator coefficients $b_0, b_1, \ldots, b_M$. Blockette [58] will specify a scaling factor S_d. Then, at any frequency f (in Hz) the response is:

$$G(f) = S_d \frac{\sum_{n=0}^{N} (a_n s^n)}{\sum_{m=0}^{M} (b_m s^m)} = S_d H_c(s) \qquad (7)$$

where $s = i\,2\,\pi\,f$ if field 3 of [54] = A, and $s = i\,f$ if field 3 of [54] = B.

As in the pole-zero case, the coefficient S_d appears to be redundant, but the response should be partitioned as described above by choosing polynomial coefficients so that the ratio of polynomials have a magnitude of 1 at $f = f_s$, so that $|G(f_s)| = S_d$ at the frequency f_s (in this case specified only in Blockette [58]); the S_d specified in Blockette [58] is then the stage gain at that frequency.

If Blockette [54] is omitted, SEED will assume the ratio of polynomials equals 1.

SEED allows these combinations of blockettes for a stage of this type: [54], [58] or [58] by itself.

Analog-Digital Converter

This stage is the transition between the analog stage (for which the input units are ground behavior and the output some other analog signal, usually volts), and the purely digital stages. This stage has no frequency response (except for a possible delay between the sample-and-hold time and the time-tagging), but it does have a gain (in digital counts per analog unit in). Use Blockettes [54], [57], and [58] to specify the nature of this stage. In Blockette [54], fields 5 and 6 of Blockette [54] give the units involved; fields 7 and 10 of Blockette [54] should both be set to zero. In Blockette [57], field 4 gives the sample rate, with field 5 set to 1 to indicate that this is also the output sample rate. Fields 7 and 8 of Blockette [57] describe any empirically determined delays and applied time shifts respectively. (Use the delay field, field 7 of Blockette [57], only in this case.) In Blockette [58], field 4 gives the digitizer response (in counts/analog unit); and field 8 may be any frequency.

Note that it is acceptable (but discouraged) to combine the digitizer description with the first FIR stage. In this case, the input units would be volts and the output units would be counts.

Digital Stages

These stages operate on sampled data, and thus operate in discrete time rather than continuous time. All operations are digital, done to finite precision; however, SEED does not describe the level of precision actually used, and, for most purposes, all arithmetic is assumed to be done to infinite precision. In general, a digital stage will consist of:

1. A discrete-time filter, either FIR (finite impulse response, also called convolution filter), or IIR (infinite impulse response, also called recursive filter).
2. Resampling of the filter output to a new rate. Usually this rate is lower, in which case this operation is called decimation.
3. Time-shifting of the decimated series by assigning a time-tag to each value that corresponds not to the time at which it was computed, but to some other time. The difference between these times is the time-shift, which is usually non-positive (where the assigned time is earlier than the actual time) to minimize the phase shift introduced by the digital filter.

Coefficient Representation for Digital Stages

This type of stage is usually used to specify Finite Impulse Response (FIR) filters. In this type ($-\infty \le k \le \infty$) is convolved with the L+1 weights or coefficients $b_0, b_1, \ldots, b_L$ to produce the output series y_k:

$$y_k = \sum_{n=0}^{L} b_n x_{k-n} \qquad (8)$$

Filters of this type are specified by Blockettes [54], [57], and [58] (or, in a special case, by using only Blockette [58]). Blockette [54] contains the weights b_n as the numerator coefficients. (There are no denominator coefficients in this case.) Blockette [57] specifies the input sample rate and the decimation factor. (Use a decimation factor of 1 if the output rate equals the input rate). Blockette [58] specifies a scaling factor, S_d. The transfer function for this filter is:

$$G(f) = S_d \sum_{n=0}^{L} b_n z^{-n} = S_d \boldsymbol{H}_c(z) \qquad (9)$$

where the z-transform variable is $z = e^{2\pi i f \Delta t}$, with Δt = the input sample interval specified in Blockette [57], and f is the frequency in Hz.

Scale the coefficients b_n so that $|Hc(zs)| = 1.0$ where $z_s = e^{2\pi i fs \Delta t}$, f_s is specified in Blockette [58]. The S_d specified in Blockette [58] is then the stage gain at f_s.

If Blockette [53] is omitted, SEED will assume that the polynomial is 1.0; this would be appropriate for a pure multiplication.

Pole - Zero Representation for Digital Stages

This type of stage is usually used to specify Infinite Impulse Response (IIR) filters (those with feedback). In this type of digital filter, the input series x_k ($-\infty \le k \le \infty$) is convolved with the LB + 1 weights $b_0, b_1, \ldots, b_{LB}$; and past values of the output series y_k are convolved with the LA weights $a_1, a_2, \ldots, a_{LA}$, to produce the output value y_k:

$$y_k = \sum_{n=0}^{LB} b_n x_{k-n} - \sum_{n=1}^{LA} a_n y_{k-n} \qquad (10)$$

The transfer function of this filter is:

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1 z^{-1} + \ldots + b_{LB} z^{-LB}}{1 + a_1 z^{-1} + \ldots + a_{LA} z^{-LA}} \qquad (11)$$

where the z-transform variable is $z = e^{2\pi i f \Delta t}$, with Δt = the input sample interval specified in Blockette [57].

Specify filters of this type with Blockettes [53], [57], and [58]. (Blockette [54] could be used to provide the coefficients, but because of the loss of precision possible in this case, we recommend not using it.) Blockette [53] will specify LB zeros, $r_1, r_2, \ldots, r_{LB}$; LA poles $p_1, p_2, \ldots, p_{LA}$, and a normalization factor A_0. The transfer function for the stage is:

$$G(z) = S_d A_0 \frac{\sum_{n=1}^{LB} (z - r_n)}{\sum_{m=1}^{LA} (z - p_m)} = S_d A_0 \boldsymbol{H}_p(z) \tag{12}$$

where z is as defined above. Choose A_0 so that $A_0 \bullet | \boldsymbol{H}_p(z_n) | = 1.0$, where $z_n =. e^{2\pi i fn \Delta t}$. Here f_n is the f_n from Blockette [53], and should be equal to the f_s in Blockette [58]. The S_d, specified in Blockette [58], is then the stage gain at that frequency, with . $G(f_n). = . G(f_s).= S_d$.

The zeros rn are the solutions of the equation:

$$b_0 + b_1 z^{-1} + \ldots + b_{LB} z^{-LB} = 0 \tag{13}$$

while the poles pm are the solutions of:

$$1 + a_1 z^{-1} \ldots + a_{LA} z^{-LA} = 0 \tag{14}$$

If Blockette [53] is omitted, A_0 will be considered to equal 1.0 (this would be appropriate for a pure multiplication).

Decimation

Blockette [57] specifies this operation. If the input series is y_m, the output series is w_n, with:

$$w_n = y_{Ln+1}, \; n = 0, 1, 2, \ldots \tag{15}$$

where L is the decimation factor and l is the offset (both are integers). The output sample interval is L times the input sample interval.

Time-shifting

As the data stream w_n emerges from the decimator, at time t_T each term is tagged (at least implicitly) with a nominal time t_N. Blockette [57] gives the time shift $\delta = t_N - t_T$ implied by this, in seconds. The effect of this time shift is to introduce a phase shift of $e^{i 2 p f \delta}$.

Examples

In the following three examples, we will assume we have a seismometer (Stage 1) followed by a digitizer (stage 2) followed by an FIR filter (Stage 3). We will then show an example Stage 0 specification summarizing these 3 stages.

Example of Specifying an Analog Stage 1.

Suppose we have a seismometer with a natural frequency f_o of 1 Hz ± 1%, a damping factor λ = 0.7 ± 3%, and a sensitivity of 150 volts per meter per second per second at 1 Hz. The acceleration transfer function would be (ignoring any constant gains):

$$H(s) = \frac{s}{s^2 + 2\lambda\omega_0 s + \omega_0^2} \quad (16)$$

There is one zero of H (s) at s = 0.The two poles of H (s) are at:

$$s = \lambda\omega_0 \pm i\,\omega_0\sqrt{1-\lambda^2} \quad (17)$$

In our example, $\omega_{0} = 2\pi \bullet (1)$ rad/sec, $\lambda = 0.7$, so we have the poles:

$$p_1 = -4.3982 + i\,4.4871$$
$$p_2 = -4.3982 - i\,4.4871 \quad (18)$$

and the zero:

$$r_1 = 0 + i\,0 \quad (19)$$

Note that both the real and imaginary parts of p_1 and p_2 may be in error by 4%, because f_0 was ±1% and λ was ±3%. However, it is known that both parts of r_1 are exactly 0. These errors are specified in Blockette [53], along with the real and imaginary parts of the poles and zeros. For this example, Blockette [53] would be filled out as follows:

Note	Field name	Type	Length	Mask or Flags	
1	Blockette type — 053	D	3	053	
2	Length of blockette			(length in bytes)	
3	Transfer function type	A	1	A	
4	Stage sequence number	D	2	01	
5	Stage signal input units	D	3	[M/S ** 2]*	
		*NOTE: What goes here is not "M/S **2", but rather a 3 digit unit look-up code such as "004" that refers to the corresponding (field #3) code in Blockette [34], the Units Abbreviations Blockette where "M/S**2" is defined.			
6	Stage signal output units	D	3	[V]*	
		*NOTE: see note above			
7	AO normalization factor (1.0 if none)	F	12	+0.87964E+01	
		NOTE: See EXAMPLE OF CALCULATING AN ANALOG STAGE RESPONSE for information on how to calculate AO.			
8	Normalization frequency fn(Hz)	F	12	+0.10000E+01	
9	Number of complex zeros	D	3	001	
10	Real zero	F	12	+0.00000E+00	} From (19)
11	Imaginary zero	F	12	+0.00000E+00	
12	Real zero error	F	12	+0.00000E+00	} Because both parts of r1 are exactly zero
13	Imaginary zero error	F	12	+0.00000E+00	
14	Number of complex poles	D	3	002	
15	Real pole #1	F	12	-0.43982E+01	} P1 from (18)
16	Imaginary pole #1	F	12	+0.44871E+01	
17	Real pole error #1	F	12	+0.17593E+00 (= 4% of 0.43982E+01)	
18	Imaginary pole error #1	F	12	+0.17948E+00 (= 4% of 0.44871E+01)	
15	Real pole #2	F	12	-0.43982E+01	} P2 from (18)
16	Imaginary pole #2	F	12	+0.44871E+01	
17	Real pole error #2	F	12	+0.17593E+00 (= 4% of 0.43982E+01)	
18	Imaginary pole error #2	F	12	+0.17948E+00 (= 4% of 0.44871E+01)	

Note that the errors listed are positive, but represent a plus/minus (±) error expressed in the same units (either rad/sec or Hz) as the units of the real and imaginary parts of the pole listed in fields 15 and 16. Blockette [58] would be filled as follows:

Note	Field name	Type	Length	Mask or Flags
1	Blockette type — 058	D	3	058
2	Length of blockette	D	4	(length in bytes)
3	Stage sequence number	D	2	01
4	Sensitivity/gain (Sd)	F	12	+0.15000E+03
5	Frequency (Hz) (fs)	F	12	0.10000E+01 NOTE: fs = fn of Blockette [53].
6	Number of history values	D	2	00

This blockette stops here because there are no history values.

Example of Specifying a Digital Stage 2

ANALOG DIGITAL CONVERTER:

Suppose the analog-to-digital converter (ADC) we are using is a 24-bit ADC for which full scale is ±20v = $\pm 2^{23}$ counts. We use Blockettes [54], [57], and [58] to describe this stage. We assume the ADC is producing 40 samples per second. Blockette [54] would be filled out as follows:

Note	Field name	Type	Length	Mask or Flags
1	Blockette type — 054	D	3	054
2	Length of blockette	D	4	(length in bytes)
3	Response type	A	1	D
4	Stage sequence number	D	2	02
5	Signal input units	D	3	[V] (by reference)
6	Signal output units	D	3	[counts] (by reference)
7	Number of numerators	D	4	0000
	REPEAT fields 8 — 9 for the Number of numerators:			
8	Not Present			
9	Not Present			
10	Number of denominators	D	4	0000
11	Not Present			
12	Not Present			

Blockette [57] (We need this one to specify the sample rate.)

Note	Fieldname	Type	Length	Mask or Flags
1	Blockette type	D	3	057
2	Length of blockette	D	4	(Length in bytes.]
3	Stage sequence number	D	2	02
4	Input sample rate(Hz)	F	10	0.4000E+02
5	Decimation factor	D	5	00001
6	Decimation offset	D	5	00000
7	Estimated delay (seconds)	F	11	+0.0000E+00
8	Correction applied (seconds)	F	11	+0.0000E+00

Blockette [58]:

Note	Field name	Type	Length	Mask or Flags
1	Blockette type — 058	D	3	058
2	Length of blockette	D	4	(length in bytes)
3	Stage sequence number	D	2	02
4	Sensitivity/gain (Sd)	F	12	+4.19430E+05*
		*NOTE: This number equals 223 counts divided by 20 volts, in this case.		
5	Frequency (Hz) (fs)	F	12	+0.10000E+01*
		*NOTE: We have specified the sensitivity at fs = 1 Hz, the same frequency at which we specified the seismometer sensitivity and the same frequency at which we calculated the normalization constant AO in stage 1.		
6	Number of history values	D	2	00

We end the blockette here if there are no history values.

Example of Specifying a Digital Stage 3

FIR FILTER:

Suppose the ADC in stage 2 is followed by a simple running 2-point averager, and we throw away every other sample (decimate by 2) at the output of this stage 3. A simple 2- point average is an example of a FIR filter, with both coefficients equal to 0.5. Suppose further that we want the gain of this FIR filter to be 2 at 0 Hz. One way to accomplish this in a real implementation is to let both of the coefficients be equal to 1.0 instead of 0.5. (This results in a gain of 2.00 at 0 Hz, or a gain of 1.9938 at 1 Hz. See "EXAMPLE OF CALCULATING A DIGITAL STAGE RESPONSE" below for an example of calculating this gain.)

In the form of equation (8), this FIR filter may be written as

$$\begin{aligned} y_0 &= b_0 x_0 + b_1 x_{-1} = b_0 x_0, \text{ (assume } x_{-1} = 0) \\ y_1 &= b_0 x_1 + b_1 x_0, \\ &\vdots \end{aligned} \qquad (20)$$

where $b_0 = 1.0$ and $b_2 = 1.0$. The decimation process would keep y_0, throw away y_1, keep y_2, and so on. The delay of this filter would appear to be about one-half of the original sample interval of 0.025 seconds, or 0.0125 seconds (the midway point of a plot of the symmetrical coefficients).

We would specify this stage 3 FIR filter by using Blockettes [54], [57], and [58]. Since Blockette [58] needs to specify the gain separately, assuming that the coefficients listed in Blockette [54] have been normalized to produce a gain of 1.00 at $f_s = 1$ Hz, we list $b_0 = b_1 = 0.50155=1/1.9938$.

Blockette [54]

Note	Field name	Type	Length	Mask or Flags
1	Blockette type — 054	D	3	054
2	Length of blockette	D	4	[Length in bytes.]
3	Response type	A	1	D
4	Stage sequence number	D	2	03
5	Signal input units	D	3	[counts] (by reference)
6	Signal output units	D	3	[counts] (by reference)
7	Number of numerators	D	4	0002
8	Numerator coefficient #1	F	12	+0.50155E+00(b0)
9	Numerator error #1	F	12	+0.00000*
		*(error in b0 -- assume zero for accurately stored digital values.)		
8	Numerator coefficient #2	F	12	+0.50155E+00 (b1)
9	Numerator error #2	F	12	+0.00000 (error in b1)
10	Number of denominators	D	4	0000*
		*NOTE: Even though we list zero denominator coefficients for FIR filters, we assume that there is a non-zero denominator value of 1.0, to avoid division by zero, when evaluating the filter transfer function.		

Blockette [57]

Note	Field name	Type	Length	Mask or Flags
1	Blockette type — 057	D	3	057
2	Length of blockette	D	4	[Length in bytes.]
3	Stage sequence number	D	2	03
4	Input sample rate (Hz)	F	10	0.4000E+02
5	Decimation factor	D	5	00002 (we are throwing away every other sample)
6	Decimation offset	D	5	00000 (we are keeping the first sample)
7	Estimated delay (seconds)	F	11	+0.1250E-01
8	Correction applied (seconds)	F	11	-0.1250E-01*
		*NOTE: We are assuming here that the data acquisition system is time tagging the data at the output of this FIR filter in such a way as to correct for the estimated delay.		

Blockette [58]

Note	Field name	Type	Length	Mask or Flags
1	Blockette type — 058	D	3	058
2	Length of blockette	D	4	[Length in bytes.]
3	Stage sequence number	D	2	03
4	Sensitivity/gain (S_d)	F	12	+0.19938E+014
5	Frequency (Hz) (f_s)	F	12	+0.10000E+01*
		*NOTE: We are again quoting the gain at the same frequency as in previous stages. We could also have quoted the gain as 2.00 at 0 Hz, because it is within 1% of the gain at 1 Hz. (The gain is easy to calculate at 0 Hz because it is just the sum of the coefficients b_i.)		
6	Number of history values	D	2	00

Example Stage O Specification

Blockette [58] must be used to summarize the overall (stages 1 through 3 in this case) gain, or system sensitivity, at a given frequency. It is best to specify this sensitivity at the same frequencies f_s and f_n used in the previous stages. Then the stage 0 sensitivity should be equal to the product of the stage 1 through K sensitivities, if there are K stages in total.

For our 3-stage example, Blockette [58] for stage 0 should be filled in as follows:

Note	Field name	Type	Length	Mask or Flags
1	Blockette type — 058	D	3	058
2	Length of blockette	D	4	[Length in bytes.]
3	Stage sequence number	D	2	00
4	Sensitivity/gain (Sd)	F	12	+1.25439E+084
		*Note: This sensitivity is assumed to be expressed in counts per m/s * * 2; that is, in terms of output units for stage K per input units for stage 1 at fs = 1 Hz. In this case, it is equal to		

$$\underbrace{150 \frac{V}{m/s^2}}_{\text{Stage 1 gain}} \cdot \underbrace{4.19430E+05 \frac{Count}{V}}_{\text{Stage 2 gain}} \cdot \underbrace{0.19938E+01 \frac{Count}{Count}}_{\text{Stage 3 gain}} = 1.25439E+08 \frac{Count}{m/s^2}$$

Note	Field name	Type	Length	Mask or Flags	
5	Frequency (Hz) (fs)	F	12	+0.10000E+01	(21)
6	Number of history values	D	2	00	

Example of Calculating Analog Stage 1 Gain and Phase

For our 1 Hz seismometer in the stage 1 example given, we have, using the form in equation (6):

$$H_p(s) = \frac{s + 0}{(s + 4.3982\, i\, 4.4871)\,(s + 4.3982 - i\, 4.4871)} \quad (22)$$

$$A_0 = 0.87964E{+}03 \;\; @ \;\; f_n = 1 \text{ Hz} \quad (23)$$

$$S_d = 0.15000E{+}03 \;\; @ \;\; f_s = 1 \text{ Hz} \quad (24)$$

How did we find A_0? To evaluate H_p (s) at f_n = 1 Hz, we substitute for s the value $s = i\omega_n = i\, 2\, \pi\, f_n$, and then calculate the modulus of H_p $(i\, 2\, \pi\, f_n)$:

$$| H_p (i\, 2\, \pi \cdot f_n) | = \left| \frac{0 + i\, 2\, \pi \cdot fn}{[4.3982 + i\,(2\, \pi\, f_n + 4.4871)]\, [4.3982 + i\,(2\, \pi\, f_n - 4.4871)] = 0.11368} \right|_{f_n = 1} \quad (25)$$

Then

$$A_0 = \frac{1}{0.11368} = 8.79640 \quad (26)$$

Of course, equation (25) may be used to evaluate $H_p(s)$ at any frequency other than f_n. The phase of H_p (s) at f may be obtained by:

$$Ø\,(f) = \tan^{-1} \left(\prod_{n=0}^{N} \frac{Im\,(s - r_n)}{Re\,(s - r_n)} \right) - \tan^{-1} \left(\prod_{m=0}^{M} \frac{Im\,(s - p_m)}{Re\,(s - p_m)} \right) \Bigg\rfloor_{s = i\, \pi\, f} \quad (27)$$

Where "Im" denotes the imaginary part of the argument and "Re" denotes the real part.

The symbol $\rfloor$ means everything to the left of the symbol is evaluated at the equation that follows it.

Example of Calculating Digital (FIR Filter) Stage Gain and Phase

For the FIR filter in the stage 3 example above, we have $b_l = b_0 = 0.50155$ and $S_d = 1.9938$ at f_s = 1 Hz. Using the form of equation (9), the transfer function of this filter is

$$G\,(f) = S_d \sum_{n=0}^{L} b_n\, z^{-n} = S_d\, \boldsymbol{H}_c\,(z) \quad = 1.9938\,(0.50155\, z0 + 0.50155\, z\text{-}1) \quad (28)$$

So

$$\boldsymbol{H}_c\,(z) = 0.50155 + 0.50155\,(z^{-1}) \quad (29)$$

In order to evaluate $\boldsymbol{H}_c$ (z), we substitute $z = e^{\,2\, \pi\, i\, f\, \Delta t}$, where f is the frequency at which we wish to evaluate $\boldsymbol{H}_c$ (z) and Δt is the sample interval, defined as the inverse of the sample rate listed in Blockette [57] for the stage. Using f = 1 Hz and Δt =1/40 sec = 0.025 secs, we have

$$\boldsymbol{H}_c\,(e^{2\, \pi\, i\, (1)\, (.025)}) = 0.50155\,(1 + e^{-2\, \pi\, i\, (1)\, (.025)}) \quad (30)$$

Using the indentity

$$e^{i\Theta} = \cos \Theta + i \sin \Theta \tag{31}$$

Equation (30) can be written

$$Hc\Big|_{f=1} = 0.50155 \ \{1 + \cos [-2 \pi (1) (.025)] + i \sin [-2 \pi (1) (.025)]\} \tag{32}$$

So the real part of $\boldsymbol{H}_c$ at f = 1 is

$$Re(\boldsymbol{H}_c)\Big|_{f=1} = 0.50155 \ \{1 + \cos (-.05 \pi) \} = 0.996925 \tag{33}$$

Ant the imaginary part of $\boldsymbol{H}_c$ at f = 1 is

$$Im(\boldsymbol{H}_c)\Big|_{f=1} = 0.50155 \ \{ \sin (-.05\pi \} = 0.07846 \tag{34}$$

The magnitude of $\boldsymbol{H}_c$ at f = 1 is then

$$| \boldsymbol{H}_c \ |_{f=1} = \{[Re(\boldsymbol{H}_c)]_{f=1}{}^2 + [Im (\boldsymbol{H}_c)]_{f=1}{}^2\}^{1/2} = \{(.996925)^2 + (0.07846)^2\}^{1/2} = 1.00000 \tag{35}$$

So we see that the coefficients $b_0 = b_1 = 0.50155$ chosen in the example really did normalize the magnitude of $\boldsymbol{H}_c$ (z) to a value of 1.0 at f = 1 Hz.

How did we know to choose the coefficients to be $b_0 = b_1 = 0.50155$? If we express $\boldsymbol{H}_c$ (z) in equation (29) in its more general form we have:

$$\boldsymbol{H}_c (z) = b_0 + b_1 z^{-1} \tag{36}$$

Equation (30) then becomes:

$$\boldsymbol{H}_c (e^{\,i\,2\,\pi\,f\,\Delta\,t}) = b_0 + b_1{}^{-i\,2\,\pi\,f\,\Delta\,t} \tag{37}$$

and (32) becomes

$$\boldsymbol{H}_c\Big|_{f=1} = b_0 + b_1 \cos [-2 \pi f \Delta t] + \sin [-2 \pi f \Delta t]\}_{f=1} \tag{38}$$

If we then substitute in the actual FIR filter coefficient values of $b_0 = b_1 = 1$ from our example, we find that actual magnitude of $\boldsymbol{H}_c$ at f = 1 is

$$\text{Actual } | \boldsymbol{H}_c \ |_{f=1} = 1.9938 \tag{39}$$

To normalize the coefficients b_i so that the resulting $\boldsymbol{H}_c$ has a value of 1.0 at $f = f_s = 1$ then, we must divide all of the bi by this actual magnitude value in equation (39). These new values of bi are then used in Blockette [54]:

$$\text{New } b_0 = \frac{\text{Actual } b_0}{1.9938} = 0.50155$$

$$\text{New b1} = \frac{\text{Actual } b_1}{1.9938} = 0.50155 \tag{40}$$

Note that this step of normalization before entry of the coefficients into the SEED blockettes is equivalent to the introduction of the A_O normalization constant for analog stages (A_O is the inverse of $| H_p (i\,2\,\pi\,f_n) |$).

If we write equation (37) for L + 1 terms we have

$$\boldsymbol{H}_c(e^{i2\pi f\Delta t}) = b_0 + b_1 e^{-i2\pi f\Delta t} + b_2 e^{-i2\pi f\Delta t} + \ldots + b_L e^{-i2\pi L f\Delta t} \qquad (41)$$

If we now let f = 0 in equation (41), we see that the magnitude of $\boldsymbol{H}_c$ is just the sum of the coefficients:

$$\boldsymbol{H}_c(e^0) = b_0 + b_1 + \ldots b_L \qquad (42)$$